AD-A020 457

RELIABILITY ACQUISITION COST STUDY (II)

R. E. Schafer, et al

Rome Air Development Center

Prepared for:

Rome Air Development Center

November 1975

RADC-TR-75-270
Final Technical Report
November 1975

RELIABILITY ACQUISITION COST STUDY (II)

Hughes Aircraft Company

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Rome Air Development Center
Air Force Systems Command
Griffiss Air Force Base, New York 13441

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER RADC-TR-75-270	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) RELIABILITY ACQUISITION COST STUDY (II)		5. TYPE OF REPORT & PERIOD COVERED Final Technical Report
		6. PERFORMING ORG. REPORT NUMBER N/A
7. AUTHOR(s) R. E. Schafer G. T. Mead J. E. Angus		8. CONTRACT OR GRANT NUMBER(s) F30602-74-C-0139
9. PERFORMING ORGANIZATION NAME AND ADDRESS Hughes Aircraft Company Ground Systems Group Fullerton CA 92634		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 62702F 55190251
11. CONTROLLING OFFICE NAME AND ADDRESS Rome Air Development Center (RBRT) Griffiss AFB NY 13441		12. REPORT DATE November 1975
		13. NUMBER OF PAGES ~~80~~ 89
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Same

18. SUPPLEMENTARY NOTES

RADC Project Engineer:
Jerry F. Lipa (RBRT)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Riliability Cost
Reliability Estimation
Reliability Program Cost
Development Costs
Optimal Reliability Spending
Cost Estimating Relationship (CER)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Quantitative relationships between reliability expenditures and reliability in the development phase for ground electronic systems were developed with regard to three phases of the reliability program (design, parts, and evaluation).

Quantitative relationships were developed for predicting reliability costs, by phase, of the reliability program and total cost, based on commonly available

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independent variables. Prediction models were developed for achieved reliability. Reliability gain due to expenditures in each phase was studied and models developed for estimating reliability gain: total and by phase. Optimal allocation of reliability resources was investigated, models developed, and a solution found.

ABSTRACT

This report presents the results of a quantitative investigation into the relationship between reliability expenditures (costs) and reliability in the development phase for ground systems. The reliability program was divided into three phases: design, parts, and evaluation. In particular, three areas were addressed.

First, quantitative relationships were developed for predicting reliability costs, by phase, of the reliability program and total cost, based on commonly available independent variables. Second, prediction models were developed for achieved reliability. Next, reliability gain (due to expenditures in each phase) was studied and models were developed for estimating reliability gain: total and by phase. Finally, optimal allocation of reliability resources was investigated. Models were developed and a solution found.

The data base consisted of ten (10) systems of relatively recent vintage. The data were subjected to an evaluation for validity and factors affecting reliability and reliability expenditures which could only confuse the results were normalized out of the data.

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1. The objective of this study was the development of relationships capable of determining and predicting the costs attributable to reliability activities during the development phase of ground electronic system/equipment acquisition.

2. The objective was met in full. Quantitative relationships were developed for predicting reliability costs by phase (design, parts, and evaluation) of the reliability program and total cost, based on commonly available independent variables such as total number of parts, total number of analog parts, total number of digital parts, predicted MTBF, and specified MTBF. Prediction models were also developed for achieved reliability. Reliability gain due to the expenditures in each of the $\overline{R}$ program phases was studied and models were developed for estimating $\overline{R}$ gain: total and by phase. The optimal allocation of reliability resources among the three major program elements was investigated, models developed, and a solution found.

3. In developing the relationships five models were considered: linear, ln-linear, exponential, second degree, and second degree with cross-products. The models were fitted to the data using the method of least squares. The best fitting model for each of the relationships desired was found to fit the data quite accurately. The data base was comprised of 10 systems: five airborne, three ground fixed, and two ground mobile, all normalized to ground fixed equipment.

4. The relationships developed in this study will be used to provide inputs into life cycle cost models for the reliability program elements, and also to give system design engineers an estimate of what reliability they can expect on a system by the expenditure of their reliability dollars.

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SECTION 0.0 – SUMMARY

In this investigation, which was limited to ground based electronic systems, the qualitatively well-known relationships between reliability expenditures in the development phase and achieved reliability (in terms of mean-time-between failures, MTBF) were quantified.

Reliability program expenditures were divided into three phases: design, parts and evaluation. Three major areas of quantification were investigated and models developed.

First, mathematical models were developed for predicting total reliability expenditures and individual phase reliability expenditures. These models were based on readily available independent variables.

Second, reliability gain was developed for each reliability program phase and for the total reliability program. Models were then developed which predict reliability gain based on reliability expenditures. This was accomplished on a phase and a total program basis.

Finally, the optimal expenditure of reliability effort was investigated. It was found that, in the present state of the art, the major reliability effort should be expended in the evaluation (developmental and demonstration testing) phase. This result is something of a discouraging comment on the "way" reliability operates at the present time: we do not know how to expend reliability effort efficiently in the design phase. It is hoped that this will change in the future since building reliability into a product is cheaper than inspecting it in.

SECTION 1.0 – INTRODUCTION

1.1 Purpose of the Study

It is now well-known that system reliability materially affects system life cycle costs. The typical Reliability Program, with its constituent parts: the design phase, the parts phase and the evaluation phase, is a powerful, useful tool in achieving reliable systems. It is also well-known that the Reliability Program often involves considerable expenditure in the development (sometimes called the acquisition) phase. Not a great deal has been done toward quantifying the relationship between Reliability (development) Program costs and reliability. The first definitive work along these lines is ref [1]. In ref [1] such relationships were studied and quantified for airborne systems. The general purpose of this present study was to quantify the relationships between Reliability (development) Program costs and reliability factors for ground equipment.

Specifically, there were three objectives:

i) Develop prediction equations for final (achieved) MTBF in terms of reliability costs (total and phase) and parts counts, and develop prediction equations for reliability costs (total and phase) in terms of parts counts and final MTBF.

ii) Determine the reliability gain for each phase (design, parts, and evaluation) and develop prediction equations for this gain in terms of the costs of the phases.

iii) Investigate the problem of allocating reliability funds in an optimal manner among the various phases of the Reliability Program.

In the next section we give the approach used in accomplishing the aforementioned objectives.

1.2 The Study Approach

In order to accomplish the objectives listed in subsection 1.1, the following tasks were set up and accomplished. These tasks are described briefly below.

1) Build the Data Base

   Data are not easy to find because reliability costs, by phase, are difficult to obtain. An intensive search led to a data base of ten (10) ground-based systems of widely varying types and of relatively recent vintage.

2) Data Evaluation and Normalization

   The data went through a final evaluation to make certain that all the required parameters (e.g., total Reliability Program costs, phase costs, predicted MTBF, specified MTBF, phase MTBF's, parts costs, etc.) were available. The data was normalized to eliminate effects which would confuse the results. For example, (use) environmental effects were largely eliminated.

3) Select Models

A number of mathematical models were selected as possible good models for use as prediction equations. Denoting for the present, a particular dependent variable as Y and a particular independent (prediction) variable as X_i five models were tried on every data set (i.e., every model was fitted to each case). The models used were (ϵ represents the random error term):

Linear

$$Y = a_0 + a_1 X_1 + \dots + a_n X_n + \epsilon$$

Ln-Linear

$$\ln Y = \ln a_0 + a_1 \ln X_1 + \dots + a_n \ln X_n + \epsilon$$

Exponential

$$Y = e^{(a_0 + a_1 X_1 + \dots + a_n X_n + \epsilon)}$$

Second Degree

$$Y = a_0 + a_{11} X_1 + a_{12} X_1^2 + \dots + a_{n1} X_n + a_{n2} X_n^2 + \epsilon$$

Second Degree With Cross Products (example, for n = 2)

$$Y = a_0 + a_{11} X_1 + a_{12} X_1^2 + a_{21} X_2 + a_{22} X_2^2 + a_{(12)} X_1 X_2 + \epsilon$$

These models were selected because they are feasible, simple, and easy to fit (by least squares). The linear and ln-linear models were used extensively in ref [1].

4) Goodness of Fit of Models

For applications of the results of this study only one of the above models is needed for any particular set of variables ($Y, X_1, \dots, X_n$). Also, it is of interest to see, over a variety of situations, whether one particular model is invariably, or even frequently, the best fitting model.

Because of the absurdity of the assumption that any particular data set (of dependent and some independent variables) is a random sample from a multivariate normal distribution, the usual measures of goodness of fit (F test, t test and Correlation) have been abandoned. The two measures of goodness of fit which we have selected are $\bar{R}$ and R.E.. The formal definitions of these quantities are given in the next section. In words, $\bar{R}$ measures the average (arithmetic mean) absolute value of the relative (to the observed values) deviation of the observed and calculated (from the model) values of the dependent variable. R. E. measures the fraction of the unexplained variation to the total variation. The smaller the values of $\bar{R}$ and R. E. for a particular data set the better the fit. The ideal, but impossible, situation would be $\bar{R} = 0$ and R. E. $= 0$.

1.3 Definitions

C_D - Reliability design phase cost (in man-days)

C'_D - Relative reliability design phase cost, $C'_D = C_D/C_T$

C_E - Reliability evaluation phase cost (in man-days)

C'_E - Relative reliability evaluation phase cost, $C'_E = C_E/C_T$

C_P - Reliability parts phase cost (in man-days)

C'_P - Relative reliability parts phase cost, $C'_P = C_P/C_T$

C_T - Total cost of reliability program (in man-days)

G_D - Reliability gain due to reliability design effort, $G_D = \Theta_D/\Theta_I$

G_E - Reliability gain due to reliability evaluation effort, $G_E = \Theta_E/\Theta_P$

G_P - Reliability gain due to reliability parts effort, $G_P = \Theta_P/\Theta_D$

G_T - Reliability gain due to total reliability program, $G_T = G_D G_P G_E = \Theta_E/\Theta_I$

k-factor - Adjustment factor for environmental applications

N - Total number of digital and analog parts, $N = N_A + N_D$

N_A - Total number of system analog parts

N_D - Total number of system digital parts

N_{EA} - Number of system parts normalized to analog

Θ_D - Post design MTBF (in hours)

Θ_E - Post evaluation MTBF (in hours)

Θ_I - Initial system MTBF without reliability enhancement (in hours)

Θ_P - Post parts MTBF (in hours)

Θ_{pred} - Predicted MTBF (in hours)

Θ_{spec} - Specified MTBF (contractual) (in hours)

$$\bar{R} = \left(\sum_{i=1}^{n} \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right| \cdot \frac{1}{n} \right) \cdot 100$$

$$R.E. = \frac{\sum_{i=1}^{n} \left((Y_i - \hat{Y}_i)^2 \big/ (n-2) \right)}{\sum_{i=1}^{n} \left((Y_i - \bar{Y}_i)^2 \big/ (n-1) \right)}$$

$$\bar{Y}_i = \sum_{i=1}^{n} Y_i \big/ n$$

where

Y_i = the i^{th} observed value of a particular function.

$\hat{Y}_i$ = the i^{th} calculated value of a particular function (calculated from a model).

n = number of observations.

SECTION 2.0 – DATA SOURCES, COLLECTION, AND EVALUATION

2.1 Data Sources

2.1.1 Internal

Due to the proprietary nature of the data required for the generation of the data base, only sources internal to the Hughes Aircraft Company could be utilized to construct the data base. Permission to access program cost documentation and labor records was provided by the various program offices.

2.1.2 External

A literature search by the National Aeronautics and Space Administration and by the Defense Documentation Center yielded no reliability cost information that could be of use for the data base structure. Generally, such data was not detailed enough to be of use.

One report, ref. [1], was an earlier work that studied the relationships of cost versus reliability for airborne systems. This report was closely studied for possible application of the approach methodology although the actual data was not applicable.

2.2 Data Collection

2.2.1 Summary of data

The data for the systems investigated for the study comprise two categories. The first category contains system characterization and reliability data. This data gathered on each system expresses the type and function of the system, the reliability values in terms of mean time between failure (MTBF) for the contractual specified ($\Theta_0 = \Theta_{spec}$), the design predicted (Θ_{pred}), and the demonstrated (Θ_{demo}) reliability values, and the system complexity as defined by the number of total parts (N). The number of parts excludes hardware and equipment that had no direct effect on the system reliability.

The number of parts was also represented by the total number of system analog parts (N_A) and the total number of system digital parts (N_D). This data is summarized in the System Characterization Table, Table 2.2.1. The second category contains the reliability cost data. The reliability program was defined by determining the reliability program phases (see section 2.4.1) and grouping the reliability program costs under the three program phases. The reliability program phase costs are the reliability design phase costs (C_D), the reliability parts phase costs (C_P), and the reliability evaluation phase costs (C_E). The total reliability program costs (C_T) are the summation of the three reliability program costs and all cost values are expressed in man-days. This data is summarized in the Reliability Program Phase Costs Table, Table 2.2.2.

TABLE 2.2.1. System Characterization Data

System Number	Type or Application	Mean Time Between Failures (Hours)			Total Parts
		Specified	Predicted	Demonstrated	
1	Shipboard ECM System	90	98	183	42901
2	Shipboard Target Acquisition System	125	146	81 (2)	46863
3	Shipboard Radar System Solid State Receiver	500	574	300 (1)	11313
4	Submarine Fire Control Display	290	270	851	21683
5	Low Frequency Sonar with Towed Array System	182	165	264	49243
6	Complex Portable Tracking and Control Center	190	210	266	36493
7	Artillery Locating Radar	250	216	216	18469
8	2400 Bit Per Second Modem	4000	5721	2005 (1)	1031
9	Tank Fire Control Ballistics Computer System	184	369	265	1751
10	Satellite Communications System	200	217.5	173	17108

(1) Reliability Demonstration Test terminated at minimum acceptable time with no failures.
(2) Reliability Demonstration Test conducted by U.S.N. personnel.

TABLE 2.2.2. Reliability Program Phase Costs

System Number	Design Phase C_D (man-days)	Parts Phase C_P (man-days)	Evaluation Phase C_E (man-days)	Total C_T (man-days)
1	214	4962	2498	7674
2	244	5301	2196	7741
3	202	4093	2249	6544
4	207	4467	2464	7138
5	237	3580	1348	5165
6	204	7233	9262	16699
7	170	1612	530	2312
8	119	3396	1452	4967
9	272	1252	928	2452
10	148	1449	2110	3707

2.2.2 Data center

The Systems Effectiveness Department of Hughes Aircraft Company's Ground Systems Group maintains a Reliability/Maintainability data center. The data center contains all reliability/maintainability information gathered during in-house development, environmental, and field use on all Ground Systems Group programs. The data center provided all of the reliability data and system characterization information for most of the systems selected for the data base. The data was extracted from proposals, contract CDRLs, Interdepartmental Correspondences, and system final reports. The sources are ref. [2], [3], [4], [5], [6], [7], [8], [9], [10], [11].

2.2.3 Intra-company contacts

The acquisition of the cost data for the reliability program activities proved to be both tedious and time consuming. The reliability program phases (see section 2.4.1) were determined in order to first define the actual efforts involved in a reliability program and second to categorize those efforts that enhance system reliability during a particular phase of the reliability program. Once the reliability program phases were defined and the reliability tasks assigned to one of the phases, the acquisition of the cost data was concentrated to particular divisions and groups that were responsible for the individual reliability program tasks.

2.3 Data Organization

2.3.1 System characterization

Data were surveyed and gathered only on systems that conformed to inhabited fixed ground, mobile ground, or shipboard system types. All of the systems investigated and those that were used to compile the data base had reliability programs that conformed to or were modifications of MIL-STD-785A, "Reliability Program for Systems and Equipment Development and Production". This standard was a contractual requirement to insure that the system reliability would achieve the overall program objectives and meet the contractual reliability requirements.

The ten systems that comprise the data base provide a sample of varied sizes, complexities, and applications. The system types range from a small single cabinet data MODEM unit to a highly complex portable tracking and control center. The environmental applications range from fixed ground shelters for some of the systems to inside a combat tank for another system. For most of the systems the environmental stresses are less severe than what would be experienced for airborne or space applications. The systems generally have adequate ventilation, the temperatures are not excessive because spaces are inhabited, and the particular design volumes are not severely constrained so design volume is not a critical factor.

2.3.2 Program time period

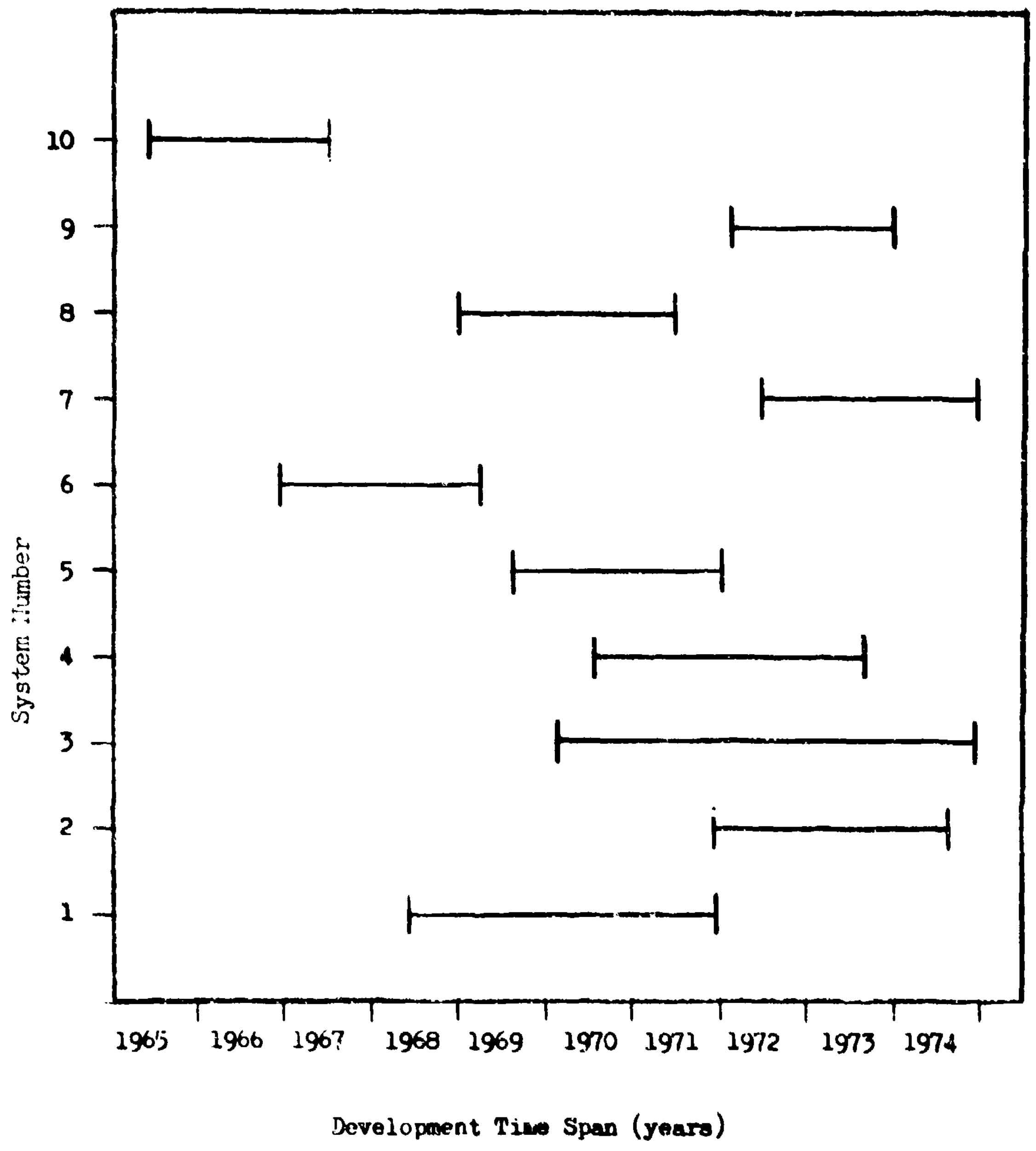
Only systems that were developed during the preceding ten years were considered as candidate systems for the data base. The development time spans for the systems are represented in the System Development Time Span table, Table 2.3.1. Although the data center contained equipment characterization and reliability data on systems that were developed as much as twenty years ago, the state of the art technology has experienced such rapid growth during that time that the system designs and individual component type reliabilities are not compatible with more current systems. Additionally, reliability cost data does not exist for very early systems due to systematic file updating and/or destruction of obsolete program files.

2.4 Data Base Categories

2.4.1 Reliability program phases

Since the overall reliability program plans differed from program to program due to system application or contractual requirements/objectives, a standard reliability program plan had to be determined so that the reliability program plan for all the data base systems would be in common terms. To simplify the standard reliability program, and yet match the available data, three categories of reliability effort were defined. Each category was a time-based phase of the reliability program where specific reliability efforts for the

TABLE 2.3.1 System Development Time Span for the Data Base



phase would reflect an improvement of the final (post development) system reliability. The reliability program phase categories are defined as follows:

1) Reliability Design Phase – the reliability tasks that comprise this phase are design review support, developing system reliability models and calculating the predicted system reliability during various stages of the program design phase, determining reliability allocations, and performing criticality studies and failure mode and effects analyses (FMEA). A system reliability model is used to define the mission use of the system, identify the operational modes of the subsystems, and model system life. The criticality study in conjunction with the FMEA identifies potential system weaknesses and critical items that significantly affect the ability of the system to successfully perform if failure occurs. The reliability prediction that is performed at various system design development phases is an indication of the achievable MTBF of the system. The prediction methodology includes the stress factors determined by the operational and environmental conditions that affect the failure rates of the system components and computes the individual failure rates up to the circuit module level. All of the module failure rates are then used in the system reliability model to determine the compatibility of the system design MTBF with the contractual MTBF. The design reviews are conducted periodically to present the results of the reliability design assessment with Engineering and other participating organizations to determine if the contractual objectives of the program are being achieved. Alternative design approaches are also assessed and tradeoffs in design are studied.

2) Reliability Parts Program Phase – The reliability tasks that make up the parts program phase are parts standardization and selection according to preferred parts lists, parts screening and vendor controls for purchased parts, component and circuit stress tests and analyses when required, and identifying and qualifying new part types. The preferred parts list maintains the number of different part types used in the equipment design to a minimum. This maximizes the usage of preferred parts with known reliability characteristics. The parts screening and vendor control tasks are directed at controlling and maintaining a high degree of part reliability throughout the development phase of a system. Component and circuit stress tests and analyses are performed if the particular operating conditions of a component or circuit are beyond the scope of a specification or derate the established reliability of a component or circuit. The qualifying of new part types when it is necessary to use a new part type in the design of a system is for the assurance of the Contractor and Engineering that the new part type will meet the reliability requirements allocated to it.

3) Reliability Evaluation Phase – the reliability tasks that describe this phase are system qualification and environmental tests, developmental tests, failure analyses, and the system reliability demonstration test. The system qualification tests are performed to determine if the system meets the operational objectives of the contract and are performed under the conditions of the intended field use of the system. The environmental tests are performed to remove infant mortality failures from the system

and demonstrate reliability growth during in-house system testing. The phenomenon of reliability growth is detailed in ref. [12]. During the system developmental testing some failures occur. These failures are thoroughly analyzed to determine the cause of failure so that, if necessary, design fixes can be implemented early in the development testing period. As design deficiencies are corrected the overall reliability of the system will improve with time. The reliability demonstration tests are performed to a MIL-STD-781 environment to determine the achieved system reliability as a result of all of the reliability program phase contributions. The primary purpose of the reliability demonstration test is to demonstrate to the customer that the system meets the contractual reliability objectives. This provides incentive to the contractor to properly implement the reliability program phases as failure to comply to the contracted reliability objectives carries severe monetary penalties.

2.4.2 Relevant cost efforts

The actual costs of the reliability program phases (C_D, C_P, C_E) are those incurred to accomplish the tasks delineated in the reliability program phase descriptions. Labor records from the Systems Effectiveness Department, Radar, Communications, and Support departments at Ground Systems Group, and the Technology Support Department, Data Systems and Electro Optical Division at Aerospace Group supplied the cost data for the data base. The costs represent only those expended to achieve the contractual reliability.

2.5 Data Evaluation: Determination of Data Validity

Throughout the data acquisition phase of the study the data sources were checked to verify the validity of the data collected (e.g., the existence of competent methods of cost control and recording in the form of task effort costs records). To make a judgement as to the validity of the actual data itself poses a unique problem of definition. Each departmental organization is responsible for a specific task and is provided funds to accomplish the task. The data only reflects the fact that the objectives were met and what costs were incurred to accomplish the objectives. What the data does not reflect is how the tasks were approached, or if the means to accomplish the tasks were optimized, or if problems occurred that may have distracted the direction of efforts away from the objectives, or if any extensive changes were implemented, etc. Therefore, the validity of the data is a subjective determination, especially since the data was gathered after the fact. However, because all of the data is internal to the Hughes Aircraft Company, these effects are certainly smaller than they would be if a number of companies had been represented in the data base.

SECTION 3.0 DESCRIPTION OF DATA ANALYSES

3.1 Data Normalization

There are usually a number of independent variables that can affect a given dependent variable. In this study we will particularly be investigating, as dependent variables, reliability and cost. The important, useful independent variables are included in the regression models. However, there are several (independent) variables that may affect the dependent variables which we do not particularly want to study. These will be "normalized" out or included in the regression model. These variables are:

i) Complexity

ii) Design differences

iii) Environmental differences

iv) Time differences

v) Non-relevant cost differences

In this study complexity was measured by parts count. The variable N (total parts count) and (N_A, N_D) were included in the regression models. To be "on the safe side" the digital parts count (N_D) on each system was converted to equivalent analog parts count and a new independent variable, N_{EA}, the total number of equivalent analog parts was included in the regression models. The conversion factor used was a constant $2 \times N_D = N_A$ (two times the number of digital parts equals their equivalent number of analog parts).

The design effects are considered to be negligible because all ten (10) systems in the data base were designed by the Hughes Aircraft Company, nine (9) of these systems were designed at Hughes-Fullerton.

The environmental differences were removed from the MTBF's by normalization. This normalization was important only when predicted or specified MTBF was used as a dependent variable. Also, when ratios (say $\Theta_{spec}/\Theta_{pred}$) were used as dependent variables the normalization factor was not needed because it cancelled out. The basis for the normalization is given in Table 3.1.1. The k-factors were developed using the fixed ground environment as the normal (k factor = 1) case. The k-factors are composites of the various environmental factors given in MIL-HDBK-217B for the various parts. The weights assigned were based on rough estimates of part distribution. It turned out that the shipboard and ground mobile factors were identical.

As can be seen from Table 2.3.1 the ten (10) systems forming the data base are of relatively recent vintage so that time effects are considered negligible. Finally, to remove non-relevant cost differences (e.g., the changing value of the dollar) all costs are measured in man-days.

TABLE 3.1.1. k-Factors for Environmental Normalization

Sys. #	Environment	θ'_{spec}	θ'_{pred}	k-factor	adjusted (θx k-factor)	
					θ_{spec}	θ_{pred}
1	Shipboard	90	98	4	360	392
2	Shipboard	125	146	4	500	584
3	Shipboard	500	574	4	2000	2296
4	Shipboard	290	270	4	1160	1080
5	Shipboard	182	165	4	728	660
6	Ground, Fixed	190	210	1	190	210
7	Ground, Fixed	250	216	1	250	216
8	Ground, Fixed	4000	5721	1	4000	5721
9	Ground, Mobile	184	369	4	736	1476
10	Ground, Mobile	200	217.5	4	800	868

3.2 The Regression Variables

The various sets of dependent and independent variables, one hundred twenty four (124) in all, are shown in Table 3.2.1. Each set was run on each of the five model types discussed in the next section with the exception that in a very few cases where there existed a large number of independent variables (e.g. 3) and a small number of data sets (as in the gain analysis) the second degree model with cross-product terms could not be run because the degrees-of-freedom were too small. Not all of the independent variable sets provided good predictions for the various dependent variables so the list in Table 3.2.1 gives all those sets tried, not the sets that were good fits.

For the 1- numbered sets the dependent variable is always cost. The independent variable(s) are those that would normally be available, at least in estimated form, early enough to be of use in predicting costs.

TABLE 3.2.1. The Regression Variables

Selection No	Dependent Variable	Independent Variable(s)
1-1	C_T	$\Theta_{spec}/\Theta_{pred}$
1-2	C_T	Θ_{spec}
1-3	C_T	Θ_{pred}
1-4	C_T	N
1-5	C_T	N_{EA}
1-6	C_T	N_A, N_D
1-7	C_D	$\Theta_{spec}/\Theta_{pred}$
1-8	C_D	Θ_{spec}
1-9	C_D	Θ_{pred}
1-10	C_D	N
1-11	C_D	N_{EA}
1-12	C_D	N_A, N_D
1-13	C_P	$\Theta_{spec}/\Theta_{pred}$
1-14	C_P	Θ_{spec}
1-15	C_P	Θ_{pred}
1-16	C_P	N
1-17	C_P	N_{EA}
1-18	C_P	N_A, N_D
1-19	C_E	$\Theta_{spec}/\Theta_{pred}$
1-20	C_E	Θ_{spec}
1-21	C_E	Θ_{pred}
1-22	C_E	N
1-23	C_E	N_{EA}

TABLE 3.2.1. The Regression Variables (Continued)

Selection No	Dependent Variable	Independent Variable(s)
1-24	C_E	N_A, N_D
1-25	C_T	$\theta_{spec}/\theta_{pred}$, N
1-26	C_T	$\theta_{spec}/\theta_{pred}$, N_{EA}
1-27	C_T	$\theta_{spec}/\theta_{pred}$, N_A, N_D
1-28	C_T	θ_{spec}, N
1-29	C_T	θ_{spec}, N_{EA}
1-30	C_T	θ_{spec}, N_A, N_D
1-31	C_T	θ_{pred}, N
1-32	C_T	θ_{pred}, N_{EA}
1-33	C_T	θ_{pred}, N_A, N_D
1-34	C_D	$\theta_{spec}/\theta_{pred}$, N
1-35	C_D	$\theta_{spec}/\theta_{pred}$, N_{EA}
1-36	C_D	$\theta_{spec}/\theta_{pred}$, N_A, N_D
1-37	C_D	θ_{spec}, N
1-38	C_D	θ_{spec}, N_{EA}
1-39	C_D	θ_{spec}, N_A, N_D
1-40	C_D	θ_{pred}, N
1-41	C_D	θ_{pred}, N_{EA}
1-42	C_D	θ_{pred}, N_A, N_D
1-43	C_P	$\theta_{spec}/\theta_{pred}$, N
1-44	C_P	$\theta_{spec}/\theta_{pred}$, N_{EA}
1-45	C_P	$\theta_{spec}/\theta_{pred}$, N_A, N_D
1-46	C_P	θ_{spec}, N

TABLE 3.2.1. The Regression Variables (Continued)

Selection No	Dependent Variable	Independent Variable(s)
1-47	C_P	Θ_{spec}, N_{EA}
1-48	C_P	Θ_{spec}, N_A, N_D
1-49	C_P	Θ_{pred}, N
1-50	C_P	Θ_{pred}, N_{EA}
1-51	C_P	Θ_{pred}, N_A, N_D
1-52	C_E	$\Theta_{spec}/\Theta_{pred}$, N
1-53	C_E	$\Theta_{spec}/\Theta_{pred}$, N_{EA}
1-54	C_E	$\Theta_{spec}/\Theta_{pred}$, N_A, N_D
1-55	C_E	Θ_{spec}, N
1-56	C_E	Θ_{spec}, N_{EA}
1-57	C_E	Θ_{spec}, N_A, N_D
1-58	C_E	Θ_{pred}, N
1-59	C_E	Θ_{pred}, N_{EA}
1-60	C_E	Θ_{pred}, N_A, N_D
2-1	Θ_E	C_T
2-2	Θ_E	N
2-3	Θ_E	N_{EA}
2-4	Θ_E	N_A, N_D
2-5	Θ_E	C_T, N
2-6	Θ_E	C_T, N_{EA}
2-7	Θ_E	C_T, N_A, N_D
2-8	Θ_E	C_E
2-9	Θ_E	C_E, N

TABLE 3.2.1. The Regression Variables (Continued)

Selection No	Dependent Variable	Independent Variable(s)
2-10	Θ_E	C_E, N_{EA}
2-11	Θ_E	C_E, N_A, N_D
2-12	Θ_E (adj)	C_T
2-13	Θ_E (adj)	N
2-14	Θ_E (adj)	N_{EA}
2-15	Θ_E (adj)	N_A, N_D
2-16	Θ_E (adj)	C_T, N
2-17	Θ_E (adj)	C_T, N_{EA}
2-18	Θ_E (adj)	C_T, N_A, N_D
2-19	Θ_E (adj)	C_E
2-20	Θ_E (adj)	C_E, N
2-21	Θ_E (adj)	C_E, N_{EA}
2-22	Θ_E (adj)	C_E, N_A, N_D
3-1	Θ_I	N_{EA}
3-2	Θ_D	Θ_I, C_D
3-3	Θ_P	Θ_D, C_P
3-4	Θ_E	Θ_P, C_E
3-5	G_D	C_D
3-6	G_P	C_P
3-7	G_E	C_E
3-8	G_T	C_T
3-9	Θ_I	N
3-10	Θ_I	N_A, N_D

TABLE 3.2.1. The Regression Variables (Continued)

Selection No	Dependent Variable	Independent Variable(s)
3-11	Θ_I	N, C_T
3-12	Θ_I	N_{EA}, C_T
3-13	Θ_I	N_A, N_D, C_T
3-14	Θ_D	N, C_T
3-15	Θ_D	N_{EA}, C_T
3-16	Θ_D	N_A, N_D, C_T
3-17	Θ_D	N, C_D
3-18	Θ_D	N_{EA}, C_D
3-19	Θ_D	N_A, N_D, C_D
3-20	Θ_P	N, C_T
3-21	Θ_P	N_{EA}, C_T
3-22	Θ_P	N_A, N_D, C_T
3-23	Θ_P	N, C_P
3-24	Θ_P	N_{EA}, C_P
3-25	Θ_P	N_A, N_D, C_P
3-26	Θ	N, C_T
3-27	Θ_E	N_{EA}, C_T
3-28	Θ_E	N_A, N_D, C_T
3-29	Θ_E	N, C_E
3-30	Θ_E	N_{EA}, C_E
3-31	Θ_E	N_A, N_D, C_E
3-32	Θ_D	C'_D
3-33	Θ_P	C'_P

TABLE 3.2.1. The Regression Variables (Continued)

Selection No	Dependent Variable	Independent Variable(s)
3-34	Θ_E	C'_E
3-35	G_D	C'_D
3-36	G_P	C'_P
3-37	G_E	C'_E
3-38	G_T	C_T, N
3-39	G_T	C_T, N_{EA}
3-40	G_T	C_T, N_A, N_D
3-41	G_T	C_D, C_P, C_E
3-42	G_T	C'_D, C'_P, C'_E

The 2- numbered sets are those with the final, achieved MTBF, Θ_E, as the dependent variable. The natural independent variables are cost and parts count and these are the variables that are used.

The 3- numbered sets form the foundation for the reliability gain and optimal allocation analysis. The dependent variables are, in turn, phase MTBF's, phase gains, and total gain. Among the independent variables, C'_D, C'_P and C'_E require special mention. These are the relative costs of the three periods and are useful in the allocation problem. For example, consider 3-42. For a fixed total gain G_T, the variables C'_D, C'_P and C'_E can be regressed on G_T and the coefficients estimated. This regression equation can then be used in attacking the optimal allocation problem.

3.3 Description of the Models and Measures of Fit

Each of the following five (5) models was fitted to each of the one hundred twenty four (124) variable sets described in the previous section. It was not expected, nor even desired, that all models would fit all the variable sets. Nor was it desired that there be at least one good model fit on every variable set. All that is needed is that there be at least one good model for each dependent variable of interest. For example, in Table 3.2.1, the 1- numbered variable sets include fifteen (15) with C_T as the dependent variable. As a worst case we only need one model to be a good fit to one of these fifteen variable sets with C_T as the dependent variable. The five models used are (ϵ represents the random error term):

Linear

$$Y = a_0 + a_1X_1 + \ldots + a_nX_n + \epsilon$$

Ln-Linear

$$\ln Y = \ln a_0 + a_1 \ln X_1 + \ldots + a_n \ln X_n + \epsilon$$

Exponential

$$Y = e^{(a_0 + a_1X_1 + \ldots + a_nX_n + \epsilon)}$$

Second Degree

$$Y = a_0 + a_{11}X_1 + a_{12}X_1^2 + \ldots + a_{n1}X_n + a_{n2}X_n^2 + \epsilon$$

Second Degree With Cross Products (example, for n = 2)

$$Y = a_0 + a_{11}X_1 + a_{12}X_1^2 + a_{21}X_2 + a_{22}X_2^2 + a_{(12)}X_1X_2 + \epsilon$$

The measures of fit for these five models are $\overline{R}$ and R.E.. These measures are described informally in section 1.2 and formally in section 1.3.

3.4 Description of Model Estimation Programs

3.4.1 General description

The reliability acquisition cost study software routines utilized for the model estimation analysis of the study data base were constructed utilizing "canned routines," i.e., programs existing in Hughes software library, and new software routines were used to merge and sort the outputs of the "canned routines" into condensed data sets. The routines for each model examined were the same except for the input data formatting. All software was programmed in FORTRAN IV utilizing the Hughes computer installation consisting of an IBM 370-165 with extensive library modules.

3.4.2 IBM routines

The "canned routines" used are from the IBM Scientific Subroutine Package for a multiple linear regression program. The multiple linear regression program consists of a main routine named REGRE, a special user supplied input subroutine named DATA, and four other subroutines from the Scientific Subroutine package: CORRE, ORDER, MINV, and MULTR. With the exception of the main routine REGRE, all of the subroutines were available in load module form only and could not be accessed. However, the formatting of the main routine REGRE could be accessed enabling the output data from the multiple linear regression program, in this case the intercept term, the regression coefficient(s), and the multiple correlation coefficient for each model selection to be written into a new data set.

3.4.3 Multiple linear regression methodology

Given a linear equation in two variables, $Y = \alpha + \beta X$ where α is the Y intercept and β is the slope of the line, the problem of finding the "best fit" line to a given set of N points (x_1, y_1), (x_2, y_2), $\cdots$, (x_N, y_N) is to determine the values a and b so that the sum of the squares of the difference between the estimated values of Y (given by $\hat{Y} = a + bX$) and the observed values of Y is a minimum. This is the least squares approach.

The constants a and b of the equation $\hat{Y} = a + bX$ are solutions of two linear equations called normal equations.

$$aN + b\sum_{i=1}^{N} X_i = \sum_{i=1}^{N} Y_i \qquad (3.4.1)$$

$$a\sum_{i=1}^{N} X_i + b\sum_{i=1}^{N} X_i^2 = \sum_{i=1}^{N} X_i Y_i . \qquad (3.4.2)$$

The constants a and b are given by

$$b = \frac{N\sum_{i=1}^{N} X_i Y_i - \sum_{i=1}^{N} X_i \sum_{i=1}^{N} Y_i}{N\sum_{i=1}^{N} X_i^2 - \left(\sum_{i=1}^{N} X_i\right)^2} \qquad (3.4.3)$$

and

$$a = \overline{Y} - b\overline{X} \qquad (3.4.4)$$

where $\overline{X}$ is the mean of the X-values and $\overline{Y}$ is the mean of the Y-values.

In the multivariable case the normal equations are similar to the linear case. The Y-values are the dependent variables, i.e., in the analysis of the data the dependent variables can be reliability phase cost or total cost, system MTBF's, or reliability gain, and the X-values are the independent variables, i.e., number of system parts, reliability phase costs, system MTBF's, or reliability gain, depending on the relationship that is being analyzed. The relationships of dependent and independent variable(s) are defined by the selection numbers in the Regression Variables Table, Table 3.2.1.

3.4.4 Data sets

The data sets were created from the study data base according to model type. The data sets were input to the appropriate model routine and the outputs from each model routine were merged into one final data set which was sorted for best fit parameters by model. The model estimation procedure is shown in Diagram 1.

3.4.5 Data sort

New software was created to calculate the deviation (error) from the regression analysis data. The measures of "goodness of fit" (R.E. and $\overline{R}$, defined in Section 1.3) were determined for each selection for all models and sorted for the "best fit" by selection number.

Model Estimation Procedure

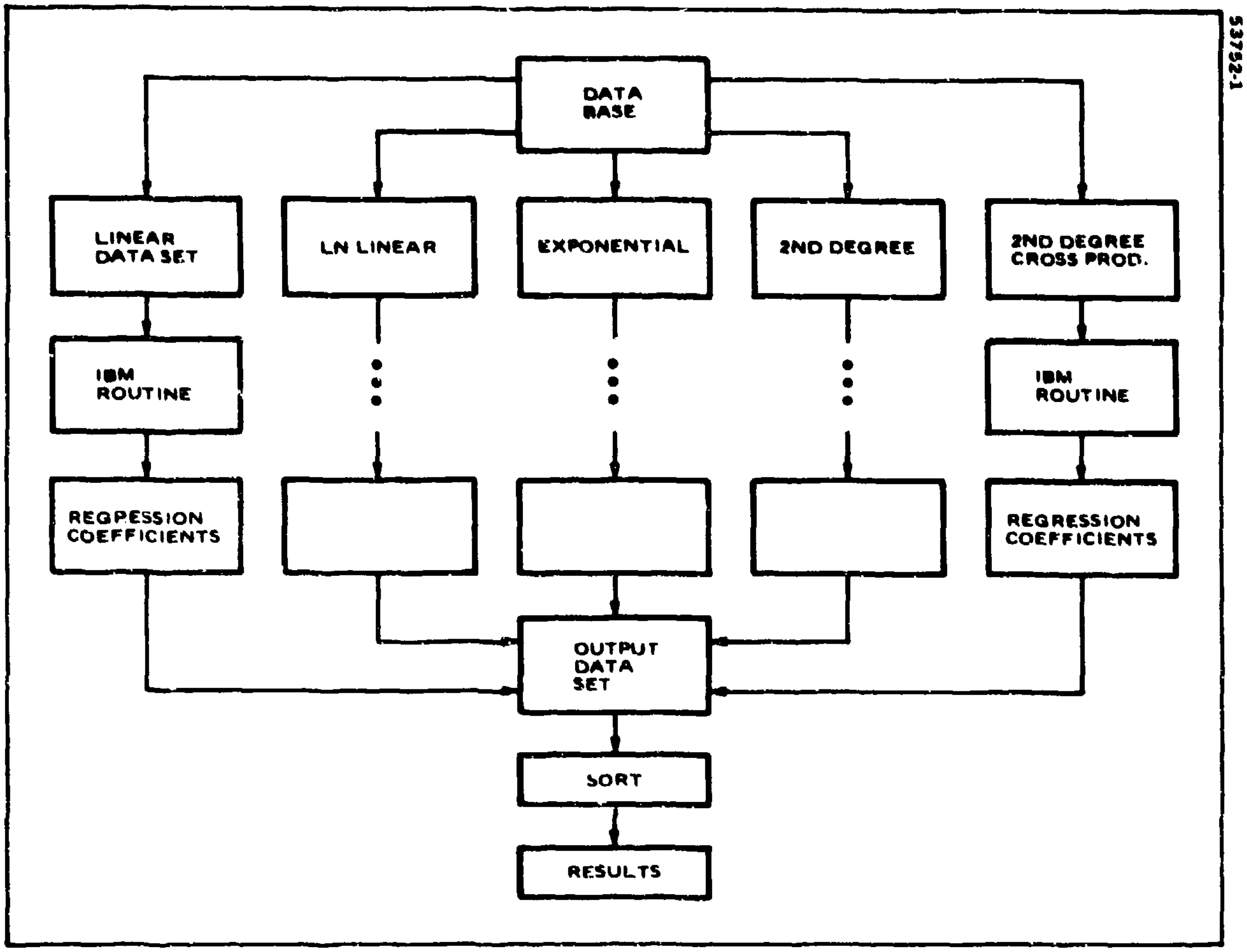


Diagram No. 1

SECTION 4.0 RESULTS OF DATA ANALYSES

4.1 The Prediction Equations for Reliability Costs

The variable sets numbered 1-, as previously mentioned in Section 3.2, all have reliability cost, in some form or other, as the dependent variable. The distribution of the sixty (60) sets (selections) is

Selection Numbers	Dependent Variable
1-1 through 1-6	C_T
1-25 through 1-33	C_T
1-7 through 1-12	C_D
1-34 through 1-42	C_D
1-13 through 1-18	C_P
1-43 through 1-51	C_P
1-19 through 1-24	C_E
1-52 through 1-60	C_E

The results of the model fits are given, by model, in Table 4.1.1. In the following sections we discuss the best fit results for each individual cost category and total costs.

4.1.1 Results for total cost, C_T

The results of the best fit for C_T as a dependent variable are given in Table 4.1.2. It is clear from this table and Table 3.2.1 that no <u>one</u> independent variable is a good predictor of C_T. Far and away the best fits are the 2nd degree cross products model with the independent variables Θ_{spec}, N_A, N_D (1-30) and Θ_{pred}, N_A, N_D (1-33). These equations are given below.

$$C_T = -24{,}814 + 41.30\,\Theta_{spec} - 0.0081\,\Theta^2_{spec} + 1.89N_A - 0.000015N_A^2$$
$$+ 0.40N_D + 0.0000082N_D^2 - 0.0013N_A\Theta_{spec} - 0.0032N_D\Theta_{spec}\,. \quad (1)$$

$$C_T = -28{,}154 + 24.85\Theta_{pred} - 0.0032\Theta^2_{pred} + 2.34N_A - 0.000027N_A^2$$
$$+ 0.76N_D - 0.000061N_D^2 - 0.0011N_A\Theta_{pred} - 0.00064N_D\Theta_{pred}\,. \quad (2)$$

TABLE 4.1.1 Goodness of Fit Results for Reliability Cost

Selection	Linear R.E.	Linear $\bar{R}$	Ln-Linear R.E.	Ln-Linear $\bar{R}$	Expon R.E.	Expon $\bar{R}$	2nd Deg R.E.	2nd Deg $\bar{R}$	2nd Deg CP R.E.	2nd Deg CP $\bar{R}$
1-1	1.12	56.27	1.17	46.93	1.19	48.06	0.75	35.04	0.75	35.04
1-2	1.08	60.13	1.09	51.39	1.18	52.92	1.01	61.54	1.01	61.54
1-3	1.07	59.44	1.07	51.63	1.20	56.42	0.98	58.23	0.98	58.23
1-4	0.88	47.54	0.95	43.61	0.95	40.93	0.81	45.97	0.81	45.97
1-5	0.87	48.22	0.95	43.35	0.93	41.07	0.84	47.46	0.84	47.46
1-6	0.86	48.80	0.93	42.26	0.92	41.21	0.75	44.60	0.61	37.68
1-7	1.09	19.31	1.09	19.74	1.87	27.64	0.98	19.73	0.98	19.73
1-8	0.76	15.51	1.03	19.47	6.21	45.42	0.69	12.43	0.69	12.43
1-9	0.81	15.07	1.09	19.38	5.01	30.28	0.62	12.99	0.62	12.99
1-10	0.94	16.08	1.05	15.92	0.95	15.25	0.85	15.27	0.85	15.27
1-11	0.97	16.86	1.07	16.48	0.98	16.23	0.88	16.40	0.88	16.40
1-12	0.90	15.31	0.61	12.84	0.92	14.91	0.76	13.84	0.65	10.49
1-13	1.10	60.27	1.17	53.80	1.20	55.33	0.67	40.02	0.67	40.02
1-14	1.11	64.26	1.17	59.26	1.22	56.68	1.05	64.85	1.05	64.85
1-15	1.10	64.68	1.13	59.47	1.19	58.38	1.02	62.70	1.02	62.70
1-16	0.73	49.06	0.87	49.95	0.80	44.78	0.72	49.26	0.72	49.26
1-17	0.72	49.31	0.87	49.84	0.77	43.82	0.72	49.24	0.72	49.24
1-18	0.72	49.53	0.86	49.25	0.78	44.22	0.63	43.35	0.41	32.60
1-19	1.12	82.39	1.20	59.16	1.18	61.43	0.88	55.40	0.88	55.40
1-20	1.05	92.76	1.10	62.94	1.19	69.12	0.98	100.00	0.98	100.00
1-21	1.05	90.83	1.11	62.65	1.21	73.22	0.97	99.12	0.97	99.12

TABLE 4.1.1 Goodness of Fit Results for Reliability Cost (Cont)

	Linear		Ln-Linear		Expon		2nd Deg		2nd Deg CP	
Selection	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$
1-22	1.02	73.05	1.08	54.48	1.11	55.63	0.90	87.73	0.90	87.73
1-23	1.01	73.14	1.08	53.87	1.09	55.80	0.93	86.93	0.93	86.93
1-24	0.99	74.18	1.06	52.33	1.08	55.36	0.83	91.90	0.76	76.93
1-25	0.85	47.73	0.86	41.80	1.33	74.26	0.50	31.48	0.45	30.76
1-26	0.84	48.47	0.86	41.38	1.09	60.97	0.50	31.26	0.49	30.61
1-27	0.84	48.59	0.83	41.36	1.14	64.49	0.38	27.84	0.11	15.51
1-28	0.83	42.38	1.04	38.24	1.03	33.03	0.69	47.76	0.46	37.51
1-29	0.86	41.73	1.04	37.68	0.99	32.88	0.74	48.02	0.33	32.92
1-30	0.85	41.92	1.02	36.22	0.98	33.16	0.61	45.44	0.00	1.15
1-31	0.88	42.73	1.06	35.52	1.01	34.37	0.71	44.46	0.21	17.85
1-32	0.86	42.51	1.05	34.71	0.98	34.70	0.77	43.56	0.15	20.59
1-33	0.85	43.01	1.03	32.76	0.97	34.88	0.62	42.88	0.01	6.05
1-34	0.71	16.04	0.54	14.16	14.55	79.62	0.31	10.83	0.31	10.77
1-35	0.78	16.68	0.62	14.99	9.99	64.78	0.36	11.95	0.36	11.93
1-36	0.60	14.69	0.11	6.23	28.67	100.00	0.16	6.60	0.15	6.75
1-37	0.75	15.32	1.02	17.91	4.08	36.57	0.24	8.43	0.24	8.24
1-38	0.76	15.51	1.03	18.51	4.92	39.57	0.24	8.18	0.24	8.31
1-39	0.66	13.90	0.45	12.48	5.67	43.24	0.24	8.75	0.11	5.86
1-40	0.80	14.96	1.05	16.77	3.44	32.68	0.18	6.75	0.15	6.00
1-41	0.81	15.11	1.07	17.69	4.11	35.59	0.19	7.19	0.19	7.15
1-42	0.73	14.24	0.54	13.20	4.22	36.08	0.22	7.59	0.11	5.14

TABLE 4.1.1 Goodness of Fit Results for Reliability Cost (Cont)

Selection	Linear		Ln-Linear		Expon		2nd Deg		2nd Deg CP	
	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$	R.E.	$\bar{R}$
1-43	0.70	48.97	0.81	49.18	0.75	45.19	0.46	32.93	0.46	32.02
1-44	0.70	49.75	0.81	48.94	0.44	44.68	0.49	33.20	0.49	33.16
1-45	0.70	49.51	0.79	47.87	0.74	44.89	0.27	27.95	0.09	11.65
1-46	0.64	34.23	0.96	42.27	0.88	31.38	0.53	41.08	0.37	33.75
1-47	0.61	33.94	0.95	41.99	0.77	30.01	0.57	38.75	0.35	30.48
1-48	0.60	34.45	0.92	41.14	0.77	30.33	0.40	34.45	0.03	9.39
1-49	0.64	36.28	0.93	40.41	0.86	33.15	0.51	36.70	0.16	18.31
1-50	0.62	36.75	0.93	39.95	0.77	32.75	0.56	34.76	0.20	20.70
1-51	0.62	37.09	0.90	38.70	0.79	32.54	0.38	31.66	0.01	3.73
1-52	0.99	71.79	1.00	54.93	2.83	100.00	0.58	53.80	0.48	56.85
1-53	0.98	71.56	0.99	54.52	2.39	100.00	0.50	47.11	0.46	56.26
1-54	0.98	72.37	0.97	52.24	2.02	100.00	0.49	45.99	0.11	28.70
1-55	1.01	82.37	1.12	50.69	1.14	52.63	0.79	100.00	0.54	73.78
1-56	1.00	81.02	1.12	49.54	1.13	52.30	0.82	100.00	0.30	64.67
1-57	0.99	80.43	1.10	47.45	1.12	51.77	0.73	100.00	0.01	13.69
1-58	1.01	79.87	1.15	46.30	1.13	53.68	0.85	95.76	0.31	41.05
1-59	1.00	78.79	1.15	44.67	1.12	53.57	0.88	96.79	0.15	41.15
1-60	0.99	79.09	1.13	43.53	1.11	53.18	0.78	94.26	0.06	27.96

TABLE 4.1.2 Best Fit Results for C_T

Selection No.	Model	R. E.	Model	$\bar{R}$
1-1	2NDD	0.75	2NDD	35.04
1-2	2NDD	1.01	LNL	51.39
1-3	2NDD	0.98	LNL	51.63
1-4	2NDD	0.81	EXP	40.93
1-5	2NDD	0.84	EXP	41.07
1-6	2DCP	0.61	2DCP	37.68
1-25	2DCP	0.45	2DCP	30.76
1-26	2DCP	0.49	2DCP	30.61
1-27	2DCP	0.11	2DCP	15.51
1-28	2DCP	0.46	EXP	33.03
1-29	2DCP	0.33	EXP	32.88
1-30	2DCP	0.00	2DCP	1.15
1-31	2DCP	0.21	2DCP	17.85
1-32	2DCP	0.15	2DCP	20.59
1-33	2DCP	0.01	2DCP	6.05

4.1.2 Results for design cost, C_D

The best fit results for C_D given in Table 4.1.3 indicate that virtually all the independent variables are good predictors of C_D. Furthermore, inspection of Table 4.1.1 indicates that all of the models do a reasonably good job of fitting. Since, typically, a user will have various independent variables available at various times the results for the best fits are given in Table 4.1.4. A blank entry in the table indicates that the particular independent variable (row) is not used on that particular selection number (column). The mathematical form of the models may be found either in Section 1.2 or Section 3.3.

4.1.3 Results for part cost, C_P

As can be seen in Table 4.1.5 (and using Table 3.2.1) no one independent variable does well as a predictor for C_P. However, again, the sets of independent variables Θ_{spec}, N_A, N_D (1-48) and Θ_{pred}, N_A, N_D (1-51) are good predictors of C_P. These equations are given below.

$$C_P = -9,133 + 15.02\Theta_{spec} - 0.0029\Theta^2_{spec} + 1.10N_A - 0.000010N_A^2$$

$$- 1.07N_D + 0.000049N_D^2 - 0.00063N_A\Theta_{spec} - 0.00044N_D\Theta_{spec} \quad (3)$$

$$C_P = -11,603 + 10.08\Theta_{pred} - 0.0013\Theta^2_{pred} + 0.92N_A - 0.000011N_A^2$$

$$+ 0.52N_D - 0.000021N_D^2 - 0.00031N_A\Theta_{pred} - 0.00061N_D\Theta_{pred} \quad (4)$$

4.1.4 Results for evaluation cost, C_E

None of the single (independent) variable cases provided suitable fits as can be seen by using Table 3.2.1 and Table 4.1.6. Again, the independent variables Θ_{spec}, N_A, N_D and Θ_{pred}, N_A, N_D provide satisfactory fits for the 2nd degree cross products model. These equations (selections 1-57 and 1-60) are given below.

$$C_E = -17,137 + 28.17\Theta_{spec} - 0.0056\Theta^2_{spec} + 0.79N_A - 0.0000048N_A^2$$

$$+ 1.78N_D - 0.000051N_D^2 - 0.00067N_A\Theta_{spec} - 0.0031N_D\Theta_{spec} \quad (5)$$

$$C_E = -18,027 + 15.77\Theta_{pred} - 0.0021\Theta^2_{pred} + 1.49N_A - 0.000017N_A^2$$

$$+ 0.41N_D - 0.000048N_D^2 - 0.00080N_A\Theta_{pred} - 0.00014N_D\Theta_{pred} . \quad (6)$$

TABLE 4.1.3 Best Fit Results for C_D

Selection No.	Model	R. E.	Model	$\bar{R}$
1-7	2NDD	0.98	LIN	19.31
1-8	2NDD	0.69	2NDD	12.43
1-9	2NDD	0.62	2NDD	12.99
1-10	2NDD	0.85	EXP	15.25
1-11	2NDD	0.88	EXP	16.23
1-12	LNL	0.61	2DCP	10.49
1-34	2DCP	0.31	2DCP	10.77
1-35	2DCP	0.36	2DCP	11.93
1-36	LNL	0.11	LNL	6.23
1-37	2DCP	0.24	2DCP	8.24
1-38	2DCP	0.24	2NDD	8.18
1-39	2DCP	0.11	2DCP	5.86
1-40	2DCP	0.15	2DCP	6.00
1-41	2DCP	0.19	2DCP	7.15
1-42	2DCP	0.11	2DCP	5.14

TABLE 4.1.4 Model Coefficients for Design Cost, C_D

Indep. Variable \ Selection and Model	1-7 (2NDD)	1-8 (2NDD)	1-9 (2NDD)	1-10 (2NDD)	1-11 (2NDD)	1-12 (LNL)	1-34 (2DCP)	1-35 (2DCP)	1-36 (LNL)	1-37 (2DCP)	1-38 (2DCP)	1-39 (2DCP)	1-40 (2DCP)	1-41 (2DCP)	1-42 (2DCP)
Intercept	467.75	806.12	107.37	186.83	190.35	6.09	779.53	775.06	2.07	352.06	368.30	186.34	181.06	226.80	130.36
$\theta_{spec}/\theta_{pred}$	-808.36						-1370.80	-1300.00	-0.05						
$(\theta_{spec}/\theta_{pred})^2$	336.06						868.01	604.36							
θ_{spec}		0.014								0.030	0.021	0.11			
$(\theta_{spec})^2$		-0.0000002								-0.000017	-0.000014	-0.000008			
θ_{pred}			0.031										0.069	0.041	0.12
$(\theta_{pred})^2$			-0.0000077										-0.000017	-0.000018	-0.000082
N				-0.0015			0.0056			-0.0070			-0.0081		
N^2				0.000000057			-0.000000042			0.[illegible]014			0.0000010		
N_{EA}					-0.0016			0.0063			-0.0055			-0.0066	
$(N_{EA})^2$					0.000000030			-0.000000053			0.0000010			0.000000006	
N_A						0.32			0.30			0.023			0.021
$(N_A)^2$												0.0000015			-0.00000[illegible]
N_D						-0.23			-0.18			-0.004			-0.004
$(N_D)^2$												0.0000026			0.0000014
$(\theta_{spec}/\theta_{pred})\cdot N$							0.0022								
$(\theta_{spec}/\theta_{pred})\cdot N_{EA}$								0.00023							
$(\theta_{spec})\cdot N$										-0.00000036					
$(\theta_{spec})\cdot N_{EA}$											0.00000046				
$(\theta_{spec})\cdot N_A$												-0.000013			
$(\theta_{spec})\cdot N_D$												0.000044			
$(\theta_{pred})\cdot N$													-0.0000025		
$(\theta_{pred})\cdot N_{EA}$														-0.00000047	
$(\theta_{pred})\cdot N_A$															-0.0000088
$(\theta_{pred})\cdot N_D$															0.000015

PROCEDURE FOR FINDING C_D

1. Determine which of the models can be used by inspecting each column (1-N) and seeing if the information required for that model is available (i.e., for model 1-7 θ specified and θ predicted are required).

2. Of the models that can be used, locate the one with the best fit to the data base from table 4.1.3 on page 31 (the one with the lowest R.E. & $\bar{R}$).

3. Starting with the general form of the model given on page 21, substitute into the model the intercept (a_o), the coefficients from table 4.1.4, and values of the independent variables to find the value of C_D.

TABLE 4.1.5 Best Fit Results for C_P

Selection No.	Model	R. E.	Model	$\bar{R}$
1-13	2NDD	0.67	2NDD	40.02
1-14	2NDD	1.05	EXP	56.68
1-15	2NDD	1.02	EXP	58.38
1-16	2NDD	0.72	EXP	44.78
1-17	2NDD	0.72	EXP	43.82
1-18	2DCP	0.41	2DCP	32.60
1-43	2DCP	0.46	2DCP	32.62
1-44	EXP	0.44	2DCP	33.16
1-45	2DCP	0.09	2DCP	11.65
1-46	2DCP	0.37	EXP	31.38
1-47	2DCP	0.35	EXP	30.01
1-48	2DCP	0.33	2DCP	9.39
1-49	2DCP	0.16	2DCP	18.31
1-50	2DCP	0.20	2DCP	20.70
1-51	2DCP	0.00	2DCP	3.73

TABLE 4.1.6 Best Fit Results for C_E

Selection No.	Model	R. E.	Model	$\bar{R}$
1-19	2NDD	0.88	2NDD	55.40
1-20	2NDD	0.98	LNL	62.94
1-21	2NDD	0.97	LNL	62.65
1-22	2NDD	0.90	LNL	54.48
1-23	2NDD	0.93	LNL	53.87
1-24	2DCP	0.76	LNL	52.33
1-52	2DCP	0.48	2NDD	53.80
1-53	2DCP	0.46	2NDD	47.11
1-54	2DCP	0.11	2DCP	28.70
1-55	2DCP	0.54	LNL	50.69
1-56	2DCP	0.30	LNL	49.54
1-57	2DCP	0.01	2DCP	13.69
1-58	2DCP	0.31	2DCP	41.05
1-59	2DCP	0.15	2DCP	41.15
1-60	2DCP	0.06	2DCP	27.96

Examples (The systems used in the examples were randomly selected)

1) **Predicting total cost C_T:**

System No. 6 has

Θ_{spec} = 190

Θ_{pred} = 210

N_A = 28,429

N_D = 8,064

C_T = 16,699.

Using equation (1) we obtain

$$C_T = -24,814 + 41.30(190) - 0.0081(190)^2 + 1.89(28,429)$$
$$- 0.000015(28,429)^2 + 0.40(8,064) - 0.0000082(8,064)^2$$
$$- 0.0013(190)(28,429) - 0.0032(190)(8,064)$$
$$= 16,466.53$$

which is in close agreement with C_T(OBS) = 16,699. Also, equation (2) leads to

$$C_T = -28,154 + 24.85(210) - 0.0032(210)^2 + 2.34(28,429)$$
$$- 0.000027(28,429)^2 + 0.76(8,064) - 0.000061(8,064)^2$$
$$- 0.0011(210)(28,429) - 0.00064(210)(8,064)$$
$$= 16,137.65.$$

2) **Predicting design cost C_D:**

System No. 1 has

Θ_{spec} = 360

Θ_{pred} = 392

N_A = 28,097

N_D = 14,804

C_D = 214.

To avoid repetition we will use just the set Θ_{spec}, N_A, N_D. From Table 4.1.4 (1-39)

$$C_D = 195.24 + 0.11(360) - 0.000032(360)^2 + 0.023(28,097) - 0.00000015$$
$$\times (28,097)^2 - 0.094(14,804) + 0.0000035(14,804)^2 - 0.000018(360)$$
$$\times 28,097 + 0.000048(360)(14,804)$$
$$= 207.71.$$

This agrees very well with the C_D(OBS) = 214.

3) Predicting part cost C_P:

System No. 3 has

$$\Theta_{spec} = 2,000$$
$$\Theta_{pred} = 2,296$$
$$N_A = 9,461$$
$$N_D = 1,852$$
$$C_P = 4,093.$$

Equation (3) gives

$$C_P = -9,133 + 15.02(2,000) - 0.0029(2,000)^2 + 1.10(9,461)$$
$$- 0.000010(9,461)^2 - 1.07(1,852) + 0.000049(1,852)^2$$
$$- 0.00063(2,000)(9,461) - 0.00046(2,000)(1,852)$$
$$= 3,454.80.$$

This result agrees well with C_P(OBS) = 4,093.

4) Predicting evaluation cost C_E:

System No. 4 has

$$\Theta_{spec} = 1,160$$
$$\Theta_{pred} = 1,080$$

$$N_A = 19{,}567$$

$$N_D = 2{,}116$$

$$C_E = 2{,}464.$$

Using Equation (5)

$$\begin{aligned} C_E &= -17{,}137 + 28.17(1{,}160) - 0.0056(1{,}160)^2 + 0.79(19{,}567) \\ &\quad - 0.0000048 \times (19{,}567)^2 + 1.78(2{,}116) - 0.000051(2{,}116)^2 \\ &\quad - 0.00067(1{,}160)(19{,}567) - 0.0031 \times (1{,}160)(2{,}116) \\ &= 2{,}346.55. \end{aligned}$$

4.2 The Prediction Equations for Final MTBF Θ_E

The "final" MTBF, Θ_E, is a fundamental part of the gain analysis to be treated in Section 4.3. The calculation of Θ_E is discussed in detail in that section. Here, it is sufficient to present the ideas in words. The final MTBF, Θ_E, refers to the mean time between failures achieved as a result of the developmental program. The time span used to compute Θ_E includes the demonstration test time even though, typically, the demonstration test is relatively (relative to the developmental testing process) short. Also, Θ_E must be computed with extreme caution. For example, it is not sufficient to compute Θ_E as the total test time (developmental and demonstration) divided by the total number of failures experienced in this time. This calculation will bias Θ_E to the low side because many of the failures are nonrecurring (e.g., design correctable) and should not be counted in a realistic assessment of an achieved MTBF. Thus the Θ_E's used were computed from the equation for the instantaneous MTBF given in Section 4.3.

The best fit results are given in Table 4.2.1. Selections Nos. 2-1 through 2-11 have Θ_E as the dependent variable while 2-12 through 2-22 have Θ_E, normalized for environment, as the dependent variable. We will use only equations from the sets 2-1 through 2-11. Although good fits are obtained in 2-12 through 2-22 it is not hard to understand why the fits are not as good as the 2-1 through 2-11 sets: although the various systems had different use environments most of the developmental and demonstration test time was under identical conditions for all the systems. Thus, the normalization of Θ_E is simply not proper.

TABLE 4.2.1 Best Fit Results for Final MTBF, Θ_E

Selection	Model	R.E.	Model	$\bar{R}$
2-1	2NDD	0.13	2NDD	4.37
2-2	2NDD	0.30	2NDD	7.19
2-3	2NDD	1.15	2NDD	12.12
2-4	LIN	1.11	LNL	10.57
2-5	LIN	0.01	LNL	1.26
2-6	LIN	0.00	LIN	0.87
2-7	LIN	0.00	LIN	0.89
2-8	LNL	0.11	LNL	3.88
2-9	LNL	0.09	LNL	3.62
2-10	LNL	0.08	LNL	3.38
2-11	LNL	0.00	LNL	0.81
2-12	2NDD	0.06	2NDD	4.05
2-13	2NDD	0.76	2NDD	26.67
2-14	2NDD	1.13	2NDD	33.58
2-15	LNL	1.24	LNL	36.24
2-16	LIN	0.27	LIN	10.58
2-17	LIN	0.29	LIN	11.41
2-18	LIN	0.20	LIN	11.03
2-19	LIN	0.10	LIN	5.52
2-20	LIN	0.10	LIN	5.26
2-21	LIN	0.09	LIN	5.33
2-22	LIN	0.07	LIN	5.80

We have selected the following sets:

Selection No.	Model	Independent Variable
2-5	ln-linear	C_T, N
2-7	ln-linear	C_T, N_A, N_D
2-9	ln-linear	C_E, N
2-11	ln-linear	C_E, N_A, N_D .

These sets represent very good fits. Although ln-linear was not always the best (e.g., see selection 2-7) it is very nearly as good as the best (linear for 2-7) and so for consistency we have used the ln-linear model. The results are:

$$\text{(2-5)} \quad \Theta_E = 72.19 C_T^{0.42} / N^{0.27} \tag{7}$$

$$\text{(2-7)} \quad \Theta_E = 62.93 C_T^{0.44} / \left(N_A^{0.25} N_D^{0.034}\right) \tag{8}$$

$$\text{(2-9)} \quad \Theta_E = 34.91 C_E^{0.28} / N^{0.049} \tag{9}$$

$$\text{(2-11)} \quad \Theta_E = 23.64 C_E^{0.27} N_A^{0.076} / N_D^{0.091} . \tag{10}$$

These equations are strikingly similar in form to their counterparts developed in ref [1]. Also, there is remarkable similarity in the various "coefficients" for a given term (independent variable) among the sets.

Example: Predicting Θ_E:

System No. 2 (randomly selected) has

$N = 46,863$

$N_A = 36,581$

$N_D = 10,282$

$C_T = 7,741$

$C_E = 2,196$

$\Theta_E = 178.15$.

Equations (7), (8), (9) and (10) lead to Θ_E = 170.11, 170.72, 178.06, 196.51 respectively.

Finally, it is interesting to use the result (modified only to our notation) given in ref. [1]:

$$\Theta_E = \frac{5.36\, C_P^{1.42} C_E^{0.64}}{N_{EA}^{1.37}} \tag{11}$$

System No. 2 has N_{EA} = 57,145 and C_P = 5,301 so that

$$\Theta_E = \frac{(5.36)(5,301)^{1.42}(2,196)^{0.64}}{(57,145)^{1.37}}$$

$$= 43.58$$

It cannot be expected that the equation developed in ref [1] for airborne data could fit ground based data and this result is an illustration of this fact.

4.3 Reliability Gain Analysis

Except possibly for certain basic, minimal requirements, money is expended on Reliability programs in order to achieve reliability gains, i.e., increase reliability (which costs money) and hence obtain near optimal, or at least lower, life cycle costs. The possibility of doing this has long been known and exploited qualitatively. In this section we will explore the relationships between reliability gain on the one hand, and costs on the other. In particular we will present quantitative relationships from which gain can be assessed.

The data base for this analysis consists of systems no. 1, 2, 4, 6, and 10. This fact requires some explanation. There are gains for each phase (design, parts, evaluation). This required the calculation of a number of "initial" and "final" MTBF's as shown in Diagram 2. We had to be very particular about the quality of data in this analysis. Consequently, enough knowledge to obtain the required numbers was available only on the five systems mentioned. Actually, this small data base is not a serious limitation because i) the quality of the data is good, and ii) the number of independent variables is small and thus not all of the degrees of freedom are used.

Before giving the results of the analyses we describe the calculation of the various phase gains. All the results of the gain calculations are given in Table 4.3.1.

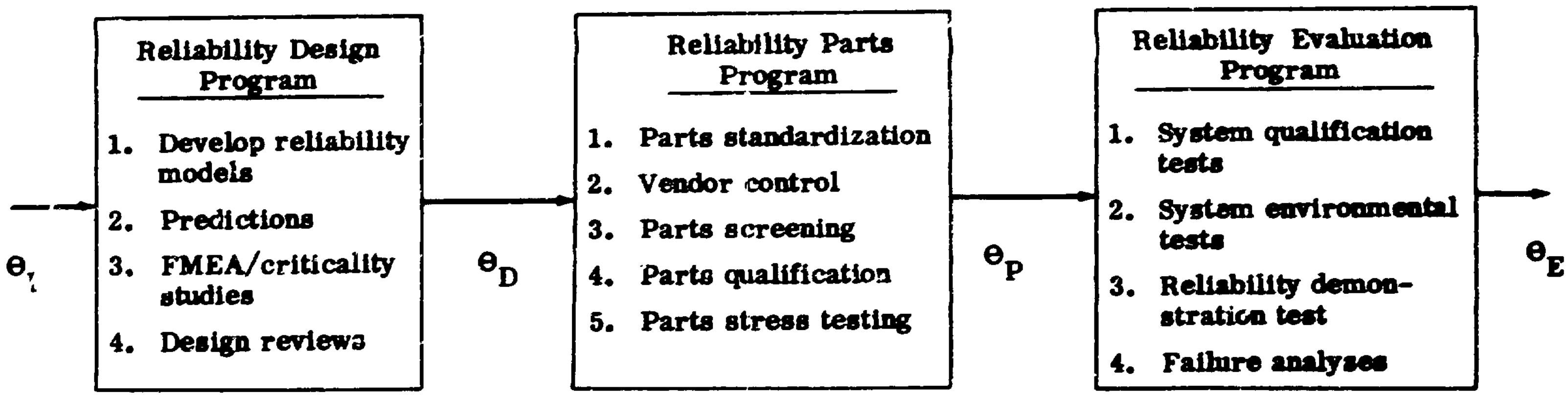


Diagram No. 2 Reliability program

TABLE 4.3.1 Reliability Gain Results

System Number	MTBF Initial θ_I	MTBF Post Design θ_D	MTBF Post Parts θ_P	MTBF Post Evaluation θ_E	Design Gain $G_D=\theta_D/\theta_I$	Parts Gain $G_P=\theta_P/\theta_D$	Test Gain $G_E=\theta_E/\theta_P$	Total Gain $G_T{}^*=\theta_E/\theta_I$
1	0.60	18.44	47.20	179.53	1.74	2.56	3.80	16.94
2	7.70	18.01	49.36	178.15	2.34	2.74	3.61	23.14
4	9.03	13.46	33.66	206.09	1.49	2.50	6.12	22.82
6	1.46	1.62	3.27	266.00	1.10	2.02	81.34	182.19
10	1.97	3.44	5.16	173.00	1.74	1.50	33.53	87.82

*Multiplication of the individual gains will not yield G_T because of round-off error.

4.3.1 Calculation of the phase gains

4.3.1.1 Evaluation gain

Generally, the final (achieved) MTBF, Θ_E, was computed by fitting the Duane model to the developmental data (for a complete description of the Duane model and fitting procedures see section 3.1 of ref. [12]). The parameter estimates were then used to compute the instantaneous MTBF*

$$\Theta_E = \mathrm{MTBF}_I = \frac{\hat{K}^{\hat{\beta}} t^{1-\hat{\beta}}}{\hat{\beta}} \tag{12}$$

The value of t used in equation (12) included the reliability demonstration test time. There were two exceptions to this calculation: systems numbered 6 and 10. Both of these systems had relatively minor but persistent design problems which were not really solved until after the developmental testing (but before reliability demonstration test). For these two systems we used Θ_E = the MTBF observed during the demonstration test.

The MTBF entering the evaluation phase (and hence leaving the part phase) is called Θ_P and was computed using the fact that for the Duane model the initial MTBF is

$$\Theta_P = \hat{K}\Gamma(1/\hat{\beta} + 1) \tag{13}$$

where $\Gamma(x)$ is the usual gamma function of x. Thus, the gain for the evaluation phase is

$$G_E = \Theta_E/\Theta_P = \frac{\hat{K}^{\hat{\beta}-1} t^{1-\hat{\beta}}}{\hat{\beta}\,\Gamma(1/\hat{\beta} + 1)} \tag{14}$$

It should be noted that the Duane model was a good fit for all five systems. The values of Θ_E, Θ_P and G_E are given in Table 4.3.1.

4.3.1.2 Parts gain

A determination of Θ_D, the system reliability before the parts program phase, was made by defining the reliability improvement of the system due to the reliability parts program. The reliability parts programs for each of the five systems examined for the gain analysis had similar parts program

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*To use cumulative MTBF would bias the results low because of the inclusion of already corrected faults.

tasks performed, i.e., parts standardization and vendor control. The identifiable differences between these similar parts program tasks were the levels of effort expended due to the individual system complexity and part types. An assessment of the similar parts program efforts, parts standardization and vendor control, for each system was made and rated as to the effectiveness of the efforts expended and a factor was determined for the gain attributable to these efforts. This gain factor determined for each system is as follows: for system numbers 1, 2, and 4 the factor is 2.5; for system number 6 the factor is 2.0; for system number 10 the factor is 1.5.

Parts screening was accomplished on two of the five gain analysis systems (system nos. 1 and 2). This effort consisted of subjecting specific part types to burn-in which qualified these parts "B level" quality.

Example: System number 2 had 1,413 devices that were subjected to parts screening out of a total system parts count of 46,863. The improvement in overall system reliability (MTBF), obtained by assessing the system failure rate with the screened parts compared to the system failure rate had the same parts not been subjected to screening, was 4.25 hours

$$\left(\text{MTBF improvement} = \frac{1}{\lambda_{\text{B level}}} - \frac{1}{\lambda_{\text{C level}}} \right)$$

or an attributable parts phase gain of 0.24. Therefore, the total reliability parts phase gain was 2.74 (2.5 + 0.24) yielding a post design MTBF (Θ_D) of 18.01 hours (49.36/2.74).

One other system, system number 6, had extensive qualification and analysis efforts expended on memory modules. This did not increase the established quality level of the memory modules as burn-in did on the parts of the two other systems, but rather was an expenditure of effort apart from burn-in and was approached differently in the parts gain analysis. System 6 had 420 memory devices. The percentage failure rate contribution to the total system failure rate for these particular devices yielded a ΔMTBF of 2.42 hours. The assessed gain attributable to the effort expended was 0.02 (1.65% improvement of 2.42 hours or MTBF improvement of 0.04 hours). This increased the parts gain (G_P) for system 6 from 2.00 to 2.02 yielding a post design MTBF (Θ_D) of 1.62 hours.

The post design MTBF (Θ_D) and the parts gain (G_P) for each of the five systems are shown in the Reliability Gain Results table, Table 4.3.1.

4.3.1.3 Design gain

The MTBF leaving the design phase, Θ_D, was computed as described in the immediately preceding subsection 4.3.1.2. In order to compute the design gain $G_D = \Theta_D/\Theta_I$ it is of course necessary to calculate the initial (entering) MTBF, Θ_I. This quantity Θ_I is an elusive quantity to say the very least: it is the MTBF that would be obtained if no money (except that money implicitly spent on reliability in the course of any program; this money could never be identified separately) were expended on reliability at all. What we did was to carefully review each of the design programs of the five systems in the data base, particularly in the proposal phase. This effort was devoted to identifying (in the proposal phase) the initial MTBF prediction, say $\Theta_{pred(i)}$. This is the initial prediction without any analysis to drive the design to meet the specified MTBF, Θ_o ($\Theta_o = \Theta_{spec}$). The results are:

System No.	$\Theta_{pred(i)}$	Θ_{spec}
1	51.8	90
2	53.4	125
4	194.6	200
6	172.7	190
10	115.0	200

We take $G_D = \Theta_{spec}/\Theta_{pred(i)}$ and hence

$$\Theta_I = \Theta_D/G_D. \tag{15}$$

The results are given in Table 4.3.1.

4.3.2 Results of the gain analyses

The gain analyses were performed on variable sets 3-1 through 3-42. The best fit results are given in Table 4.3.2. It is clear that the ln-linear model is virtually always the best (with respect to $\overline{R}$) and when it is not it is very close to the best. Further evidence is that in ref. [1] the ln-linear model was also found to fit very well. Thus we will use the ln-linear model throughout the gain analysis.

The important variable sets are:

Selection No.	Dependent Variable	Independent Variable(s)
3-5	G_D	C_D
3-6	G_P	C_P
3-7	G_E	C_E
3-8	G_T	C_T
3-35	G_D	C'_D

TABLE 4.3.2 Best Fit Results for Reliability Gain Analysis

Selection	R. E.		$\bar{R}$	
3-1	LIN	1.14	LNL	89.86
3-2	LIN	0.10	LNL	18.15
3-3	LNL	0.00	LNL	1.49
3-4	LNL	0.10	LNL	3.66
3-5	LIN	1.15	LNL	20.97
3-6	LNL	0.79	LNL	13.77
3-7	LIN	0.22	LNL	69.74
3-8	LIN	0.65	LNL	90.05
3-9	LIN	1.14	LNL	87.31
3-10	LIN	1.14	LNL	80.63
3-11	LIN	0.76	LNL	59.41
3-12	LIN	0.80	LNL	68.49
3-13	LIN	0.73	LNL	44.01
3-14	LIN	0.39	LNL	57.59
3-15	LIN	0.48	LIN	73.75
3-16	LIN	0.33	LNL	40.11
3-17	LIN	0.66	LNL	85.03
3-18	LIN	0.67	LNL	94.28
3-19	LNL	0.03	LNL	6.25
3-20	LIN	0.37	LNL	73.15
3-21	LIN	0.47	LNL	97.84
3-22	LIN	0.29	LNL	47.42
3-23	LIN	0.69	LNL	38.56
3-24	LIN	0.78	LNL	51.36
3-25	LIN	0.60	LNL	84.10
3-26	LIN	0.01	LNL	1.26
3-27	LIN	0.00	LIN	0.87
3-28	LIN	0.00	LIN	0.89
3-29	LNL	0.09	LNL	3.62
3-30	LNL	0.08	LNL	3.38
3-31	LNL	0.00	LNL	0.82
3-32	LIN	1.24	LNL	98.40
3-33	LNL	0.05	LNL	25.54
3-34	LNL	1.07	LNL	11.73
3-35	LNL	0.70	LNL	11.94
3-36	LNL	0.09	LNL	4.53
3-37	LIN	0.40	LNL	31.73
3-38	LIN	0.35	LNL	72.58
3-39	LIN	0.40	LIN	73.44
3-40	LNL	0.20	LNL	51.15
3-41	LNL	0.00	LNL	3.82
3-42	LNL	0.00	LNL	8.89

Selection No.	Dependent Variable	Independent Variable(s)
3-36	G_P	C'_P
3-37	G_E	C'_E
3-38	G_T	C_T, N
3-39	G_T	C_T, N_{EA}
3-40	G_T	C_T, N_A, N_D
3-41	G_T	C_D, C_P, C_E
3-42	G_T	C'_D, C'_P, C'_E

The independent variables C'_D, C'_P, C'_E, are the relative costs

$$
\begin{aligned}
C'_D &= C_D/C_T \\
C'_P &= C_P/C_T \\
C'_E &= C_E/C_T
\end{aligned}
\tag{16}
$$

where

$$C_T = C_D + C_P + C_E$$

and are given in Table 4.3.3 for all ten systems even though only systems 1, 2, 4, 6, and 10 comprise the gain analysis data base. The relative costs will be used in the optimal allocation analysis of section 4.4.

4.3.2.1 Total cost gain analysis

It is of no particular importance that the variable sets 3-38, 3-39 and 3-40 are relatively poor fits since the major independent variables are C_T. C_D, C_P and C_E. Thus the important sets, for G_T, are 3-8, 3-41 and 3-42.

The equation for G_T in terms of C_T is

$$G_T = 0.27\ C_T^{0.56}\ . \tag{17}$$

TABLE 4.3.3 Relative Reliability Program Phase Costs

System Number	C'_D	C'_P	C'_E
1	0.0279	0.6466	0.3255
2	0.0315	0.6848	0.2837
3	0.0309	0.6254	0.3437
4	0.0290	0.6258	0.3452
5	0.0459	0.6931	0.2610
6	0.0122	0.4331	0.5547
7	0.0735	0.6973	0.2292
8	0.0240	0.6837	0.2923
9	0.1109	0.5106	0.3785
10	0.0399	0.3909	0.5692
MEAN	0.0426	0.5991	0.3583

This fit is not very good but the fit for G_T in terms of the individual absolute and relative costs are much better; indeed quite good:

$$G_T = (2.31 \times 10^{-11}) C_D^{6.44} C_P^{-3.61} C_E^{3.01} \qquad (18)$$

$$C_D + C_P + C_E = C_T$$

$$G_T = (2.25 \times 10^{-6}) C_D'^{(-1.37)} C_P'^{(-10.39)} C_E'^{(-5.88)} \qquad (19)$$

$$C'_D + C'_P + C'_E = 1$$

This fact illustrates that while total costs affect total gain, much more important is how the money (measured in man-days in this report) is spent.

We defer an example until we have discussed the individual gain results by phase.

4.3.2.2 Design cost gain analysis

For this analysis the important variable sets are 3-5 and 3-35. These two sets are good fits and the results are:

$$G_D = 0.27\ C_D^{0.34} \tag{20}$$

$$G_D = 9.42\ C_D'^{(0.48)} \quad , \quad 0 < C_D' \leq (1 - C_P' - C_E') . \tag{21}$$

The result of equation (20) is strikingly similar to the result (in our notation) found in ref. [1]:

$$G_D = 0.30\ C_D^{0.25} .$$

No results were given in ref. [1] to permit comparison with equation (21).

4.3.2.3 Part cost gain analysis

The important variable sets here are 3-6 and 3-36. Again, the fits were quite good and the results are:

$$G_P = 0.19\ C_P^{0.29} \tag{22}$$

$$G_P = 3.87\ C_P'^{(0.91)} \quad , \quad 0 < C_P' \leq (1 - C_D' - C_E'). \tag{23}$$

The result of equation (22) is somewhat similar to the result in ref. [1], which in our notation is:

$$G_P = 1.145\ C_P^{0.137} .$$

As before, and also for G_E, no results were given in ref. [1] to permit comparison to equation (23). That is, relative costs were not investigated in ref. [1].

4.3.2.4 Evaluation cost gain analysis

The important data sets are 3-7 and 3-37 and the results are:

$$G_E = 0.000029\, C_E^{1.61} \tag{24}$$

$$G_E = 561.94\, C_E'^{(4.20)} \qquad 0 < C_E' \leq (1 - C_D' - C_P') . \tag{25}$$

Although the fit was not very good for equation (24) (the $\overline{R}$ was quite large) it is similar to the result given in ref. [1] (in our notation)

$$G_E = 0.0064\, C_E^{0.95} .$$

The results of these analyses are shown graphically in Figure 4.3.1.

<u>Example</u>

It is clear that

$$G_T = \frac{\Theta_E}{\Theta_I} = \left(\frac{\Theta_D}{\Theta_I}\right) \left(\frac{\Theta_P}{\Theta_D}\right) \left(\frac{\Theta_E}{\Theta_P}\right) = G_D G_P G_E \quad . \tag{26}$$

Thus, it will be interesting to compare the (direct) results obtained from equation (18):

$$G_T = 2.31 \times 10^{-11} C_D^{6.44} C_P^{-3.61} C_E^{3.01}$$

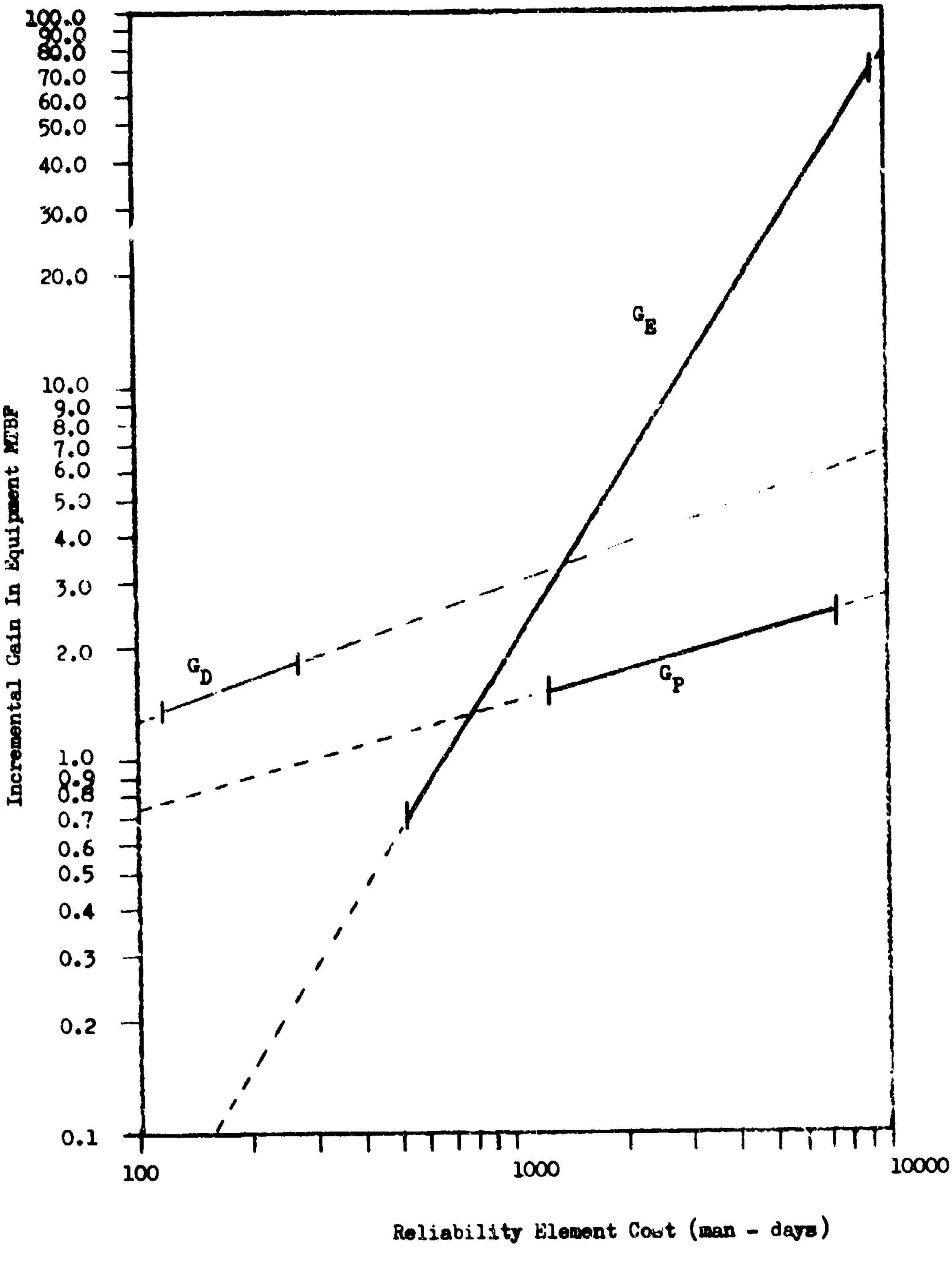
and equations (20), (22) and (24):

$$G_D = 0.27\, C_D^{0.34};\; G_P = 0.19\, C_P^{0.29};\; G_E = 0.000029\, C_E^{1.61} .$$

System No. 4 (randomly selected) has

$$C_D = 207;\; C_P = 4,467;\; C_E = 2,464.$$

FIGURE 4.3.1 Incremental Gain vs. Cost



Thus, from equation (18) we have

$$G_T = 20.45.$$

From the individual equations

$$G_D = 1.66;\ G_P = 2.17;\ G_E = 8.37$$

and

$$G_T = 30.15.$$

Both of these calculated values of G_T compare well with the actual (observed) gain for system no. 4 of 22.82. From a purely mathematical standpoint, equation (18) would be the preferred method.

4.4 Optimal Allocation Analysis

A question of some importance in allocating reliability expenditures is: for a fixed total amount of expenditure how should the effort be divided among the phases? That is, for fixed C_T, what should C_D, C_P, and C_E be where $C_D + C_P + C_E = C_T$. Since C_T itself, varies from program to program, and since

$$C'_D = \frac{C_D}{C_T};\ C'_P = \frac{C_P}{C_T};\ C'_E = \frac{C_E}{C_T},$$

it is sufficient to optimally allocate to the relative costs C'_D, C'_P and C'_E. Then for a fixed and known C_T the values C_D, C_P and C_E can be obtained from $C_D = C'_D C_T$; $C_P = C'_P C_T$; $C_E = C'_E C_T$.

To keep things simple we will use the linear model. It has a very small R. E. and a not too large $\bar{R}$ so it is a reasonable fit. We will address the best fitting case, ln-linear, shortly.

For the linear model

$$C_T = 90.36 - 3,569.25\ C'_D - 94.00\ C'_P + 309.88\ C'_E. \tag{27}$$

However, since $C'_D + C'_P + C'_E = 1$, one of the variables can be eliminated. Arbitrarily eliminating C'_E we have

$$G_T = 400.24 - 3,879.13\, C'_D - 403.88\, C'_P. \tag{28}$$

Of course it is silly, even incompetent, to allow absurdities like $C'_D = 0$, $C'_P = 0$, $C'_E = 1$ or $C'_D = 1$, $C'_P = 0$, $C'_E = 0$. There are many other absurdities like these and in order to avoid them we need additional restrictions. Using the observed values actually expended from Table 4.3.3 we establish the following additional restrictions. Generally we took (near) the maximum and minimum observed expenditure rates.

$$\left.\begin{array}{l} 0.01 \le C'_D \le 0.12 \\ \\ 0.35 \le C'_P \le 0.70 \\ \\ 0.20 \le C'_E \le 0.60 \end{array}\right\} \tag{29}$$

$$C'_D + C'_P + C'_E = 1.$$

Now we can view the problem: find C'_D, C'_P and C'_E so that G_T of equation (28) is maximized subject to the restrictions of (29), as a linear programming problem. Actually, this problem could be solved graphically but there are many "canned" linear program routines. We used the LINPRO routine written in XBASIC for the G. E. 265 computer. The results are:

$$C'_D = 0.01$$

$$C'_P = 0.39$$

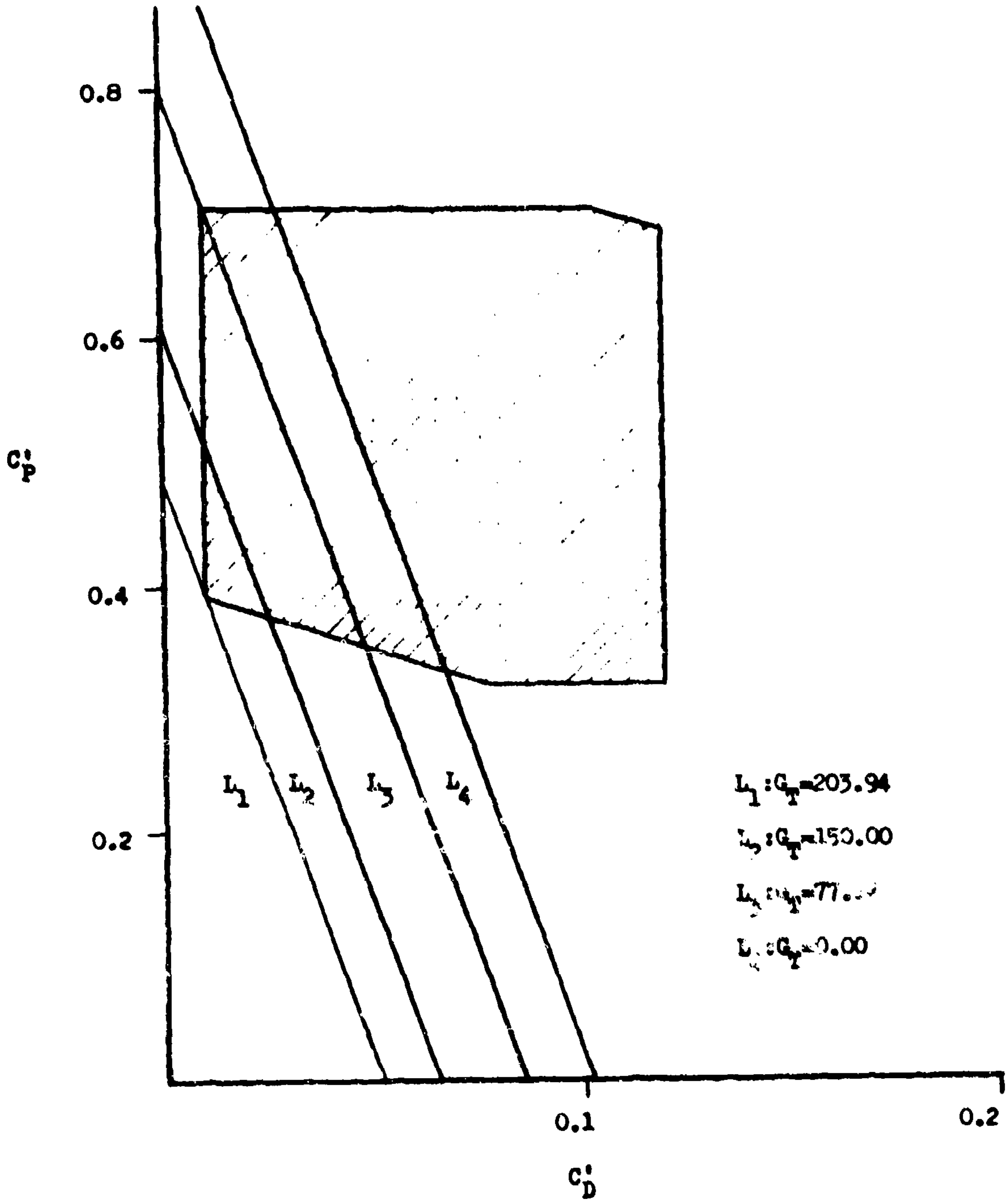
$$C'_E = 0.60$$

and

$$G_T = 203.94.$$

The region of feasible solutions and the optimal solution are shown in Figure 4.4.1.

FIGURE 4.4.1 Optimal Allocation Analysis - Linear Model for G_T



The solution is: spend as little as possible on reliability design costs and as much as possible on reliability evaluation. This is to be expected since the greatest gains were achieved in the evaluation phase. One might be tempted to think the solution is not very good because of the old phrase "it is better to build quality in rather than inspect it in"; however, it must be remembered C_D represents reliability design expenditure and not total design expenditure. Clearly, from a reliability viewpoint, the expenditures are relatively more efficiently made in the evaluation phase.

Now we address the ln-linear model for G_T since it is the best fitting model. The linear program approach will not work because, even though we can transform to linearity in the logarithm the restrictions given in (29) become non-linear. However, one of the variables can be eliminated from the equation for G_T since, as before, $C'_D + C'_P + C'_E = 1$ and we have

$$G_T = (2.25 \times 10^{-6})(1 - C'_P - C'_E)^{-1.37} C'_P{}^{(-10.39)} C'_E{}^{(-5.88)}. \tag{30}$$

In Figure, 4.4.2 we have shown G_T is a function of C'_E for various values of C'_P. The solid lines represent values of G_T that are admissible in the sense that the independent variables C'_D, C'_P, C'_E satisfy the restrictions of (29). It is clear from Figure 4.4.2, that while we have not found the absolute optimal solution, the optimal solution is near $C'_D = 0.01$; $C'_P = 0.42$; $C'_E = 0.57$. This is about the same as the solution for the linear case previously discussed. There the optimal solution was $C'_D = 0.01$; $C'_P = 0.39$; $C'_E = 0.60$.

4.5 Prediction of Individual Phase MTBF's

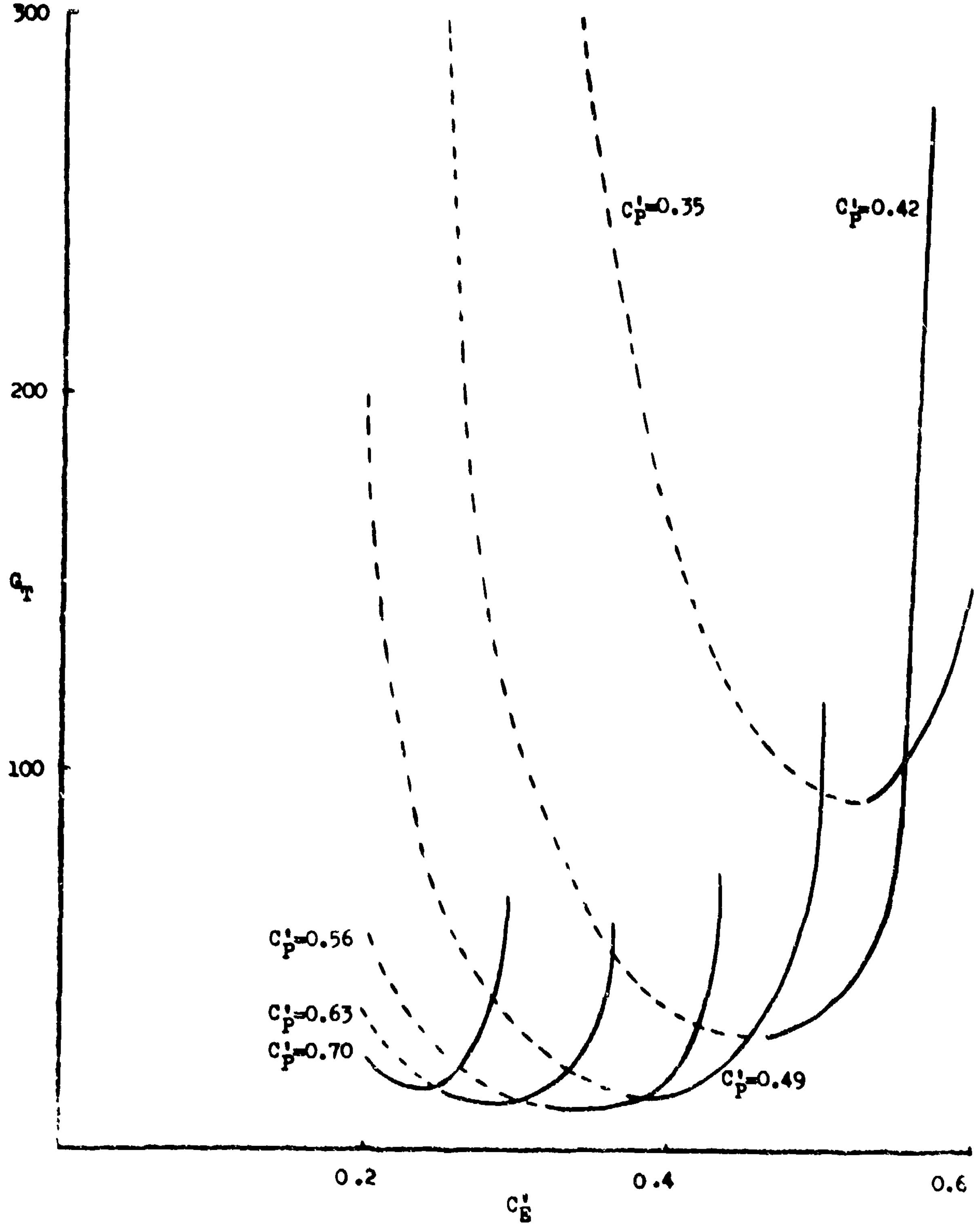
In this section we will give the equations for predicting the individual phase MTBF's based on just two independent variables in each case: the reliability expenditures in that phase and the MTBF entering that phase. The ln-linear models are the overall best fitting models for the three variables sets 3-2, 3-3, and 3-4. It should be noted that Θ_D, Θ_P and Θ_E are the absolute values of the MTBF leaving a particular phase given that the previous phase, if any, has been implemented. If, for example, the parts phase had not been implemented then the ratio (from this study) $\Theta_E/\Theta_P = \Theta^*$ (say) could be applied to Θ_D (assuming the design phase was implemented) to determine Θ_E, i.e. $\Theta_E = \Theta^*\Theta_D$. In short, omission of a phase will affect the absolute value of the MTBF leaving subsequent phases.

4.5.1 Prediction of Θ_D

In this case the independent variables are C_D and Θ_I, the ln-linear model fits well (see Table 4.3.2) and we have

$$\Theta_D = 4.94\, \Theta_I^{1.19} C_D^{-0.26}.$$

FIGURE 4.4.2 Total Gain vs. Relative Reliability Cost Elements



4.5.2 Prediction of Θ_P

The ln-linear model is an excellent fit (see Table 4.3.2) and we obtain

$$\Theta_P = 0.19\,\Theta_D^{1.15} C_P^{0.26}.$$

4.5.3 Prediction of Θ_E

The ln-linear model is again an excellent fit (see Table 4.3.2) and we obtain

$$\Theta_E = 18.64\,\Theta_P^{0.014} C_E^{0.29}.$$

<u>Example</u>: System No. 10 (randomly selected) has

$$C_E = 2,110; \; \Theta_P = 5.16; \; \Theta_E = 173$$

thus

$$\Theta_E = 18.64\,(5.16)^{0.014}\,(2,110)^{0.29}$$

$$= 174.9.$$

Using the model from ref. [1] for final MTBF (changed only to fit our notation)

$$\Theta_E = 0.094\,(5.16)^{0.68}\,(2,110)^{0.74}$$

$$= 83.8.$$

Of course, we would not expect the airborne model to fit the ground base model.

SECTION 5.0 CONCLUSIONS AND RECOMMENDATIONS

Based on the results obtained in ref. [1] for airborne systems and this present investigation for ground based systems, the relationships between reliability cost and reliability achieved can, and have been quantified. Also, the models developed in this present study, in most cases, are quite excellent fits (this is not to say the models developed in ref. [1] were not good fits). Of course we have for the reader the usual admonition that use of the models <u>outside</u> the range of the independent variables in the data base is risky. However, the risk should be minimal since we know that the variables in question are related in some fashion.

Although the comparisons we made show that the airborne models from ref. [1] cannot be used for the ground based data, the resemblance between the various models of the two studies was striking in the gain analysis. This lends further credence to the validity of the relationships and models developed.

Finally, all of the models are worth developing further, i.e., by expanding the data base. However, for special mention we single out the optimal allocation analysis of section 4.4. The data base should be expanded and the optimal allocation should be studied in much more detail since it is a powerful, very powerful, tool.

SECTION 6.0 APPENDIX: GRAPHICS OF ACTUAL VS. PREDICTED VALUES

Figure 6.1 Predicted Total Cost vs Actual Total Cost

$$C_T = -24,814 + 41.30\theta_{spec} - 0.0081\theta^2_{spec} + 1.89N_A - 0.000015N_A^2 + 0.40N_D + 0.0000082N_D^2$$
$$-0.0013N_A\theta_{spec} - 0.0032N_D\theta_{spec}$$

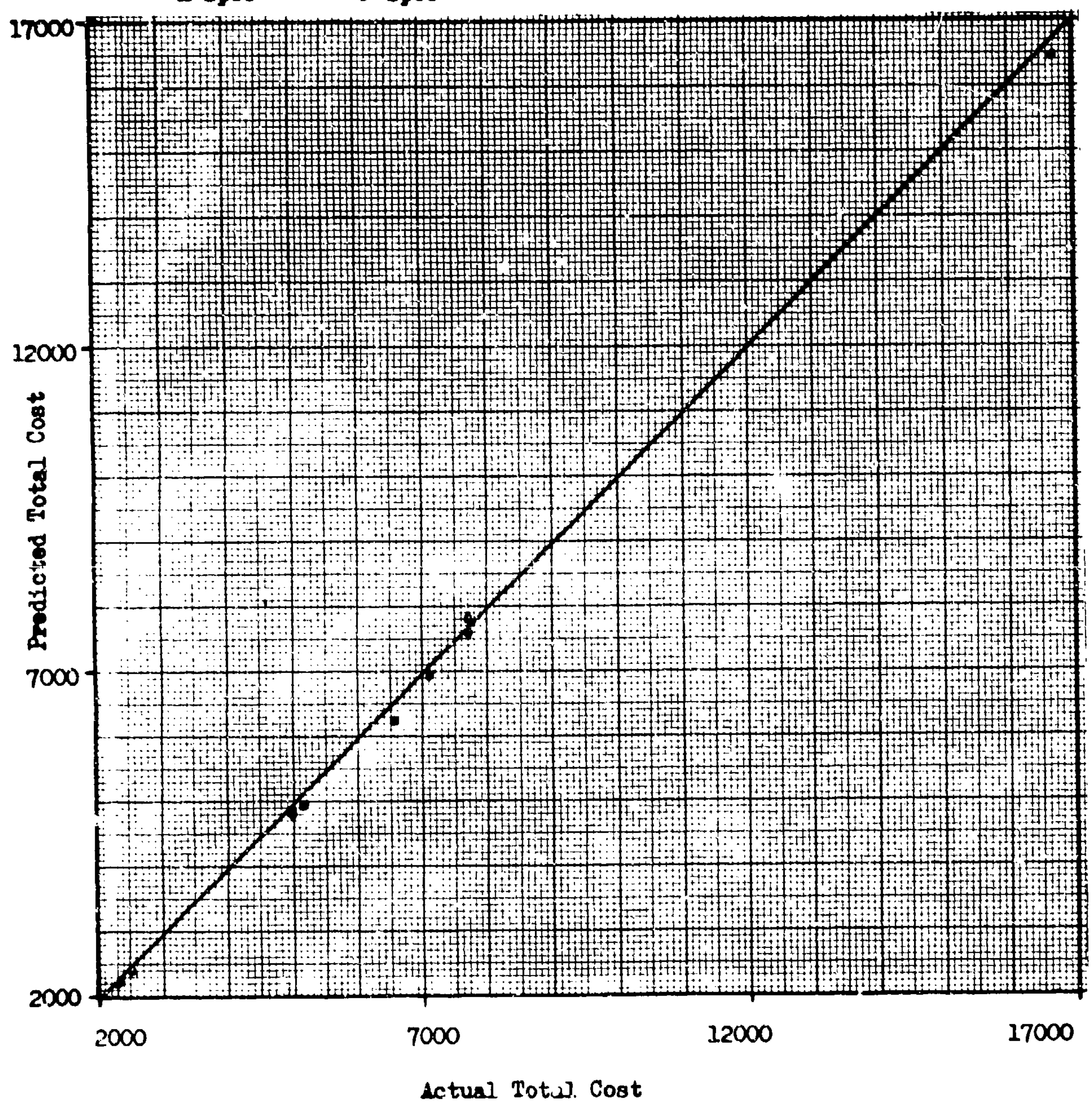


Figure 6.2 Predicted Total Cost vs Actual Total Cost

$$C_T = -28{,}154 + 24.65\theta_{pred} - 0.0032\theta^2_{pred} + 2.34N_A - 0.000027N_A^2 + 0.76N_D - 0.000061N_D^2$$
$$-0.0011N_A\theta_{pred} - 0.00064N_D\theta_{pred}$$

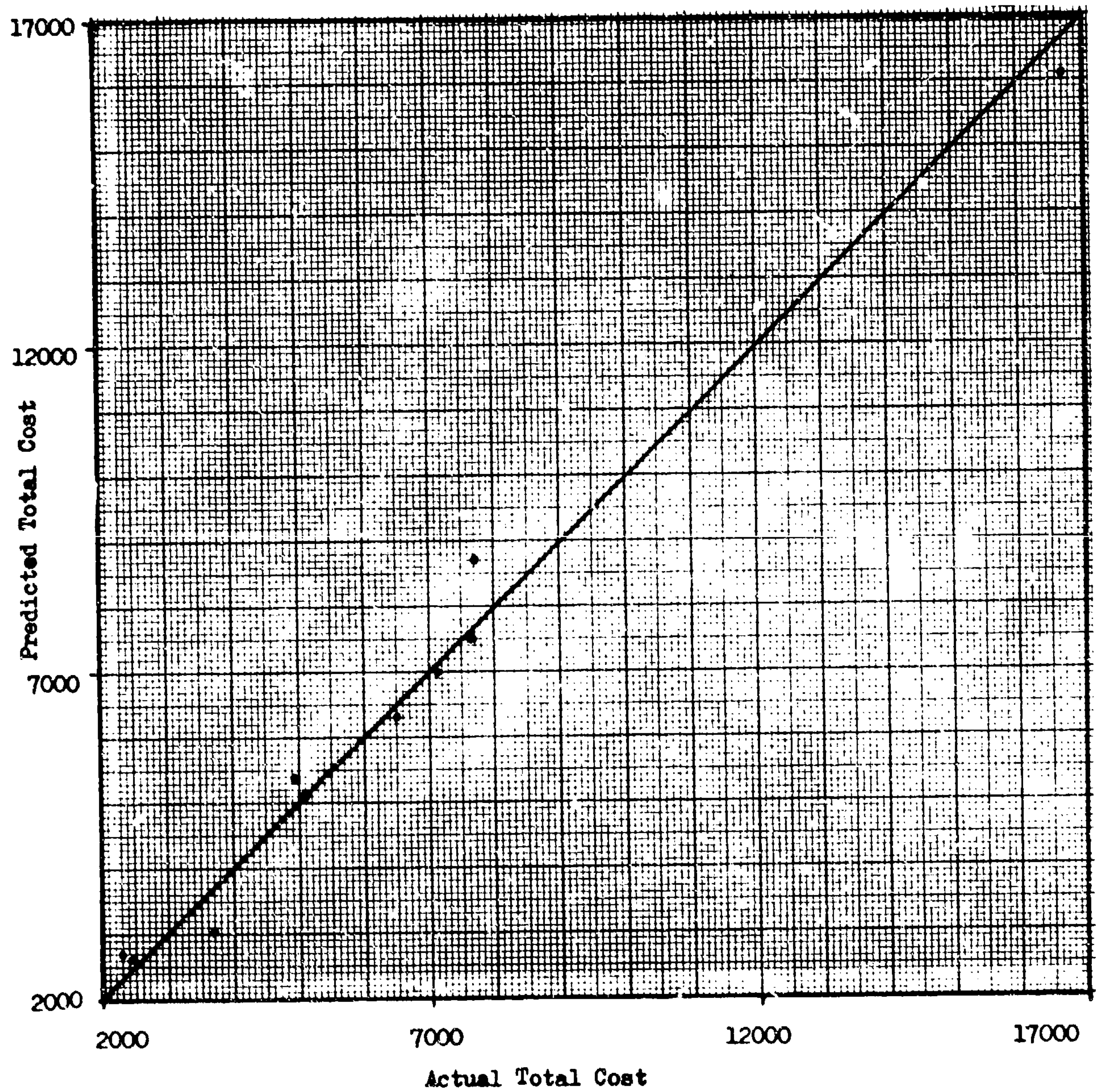


Figure 6.3 Predicted Parts Phase Cost vs Actual Parts Phase Cost

$$C_P = -9.133 + 15.02\theta_{spec} - 0.0029\theta^2_{spec} + 1.10N_A - 0.000010N_A^2 - 1.07N_D + 0.000049N_D^2$$
$$- 0.00063N_A\theta_{spec} - 0.00044N_D\theta_{spec}$$

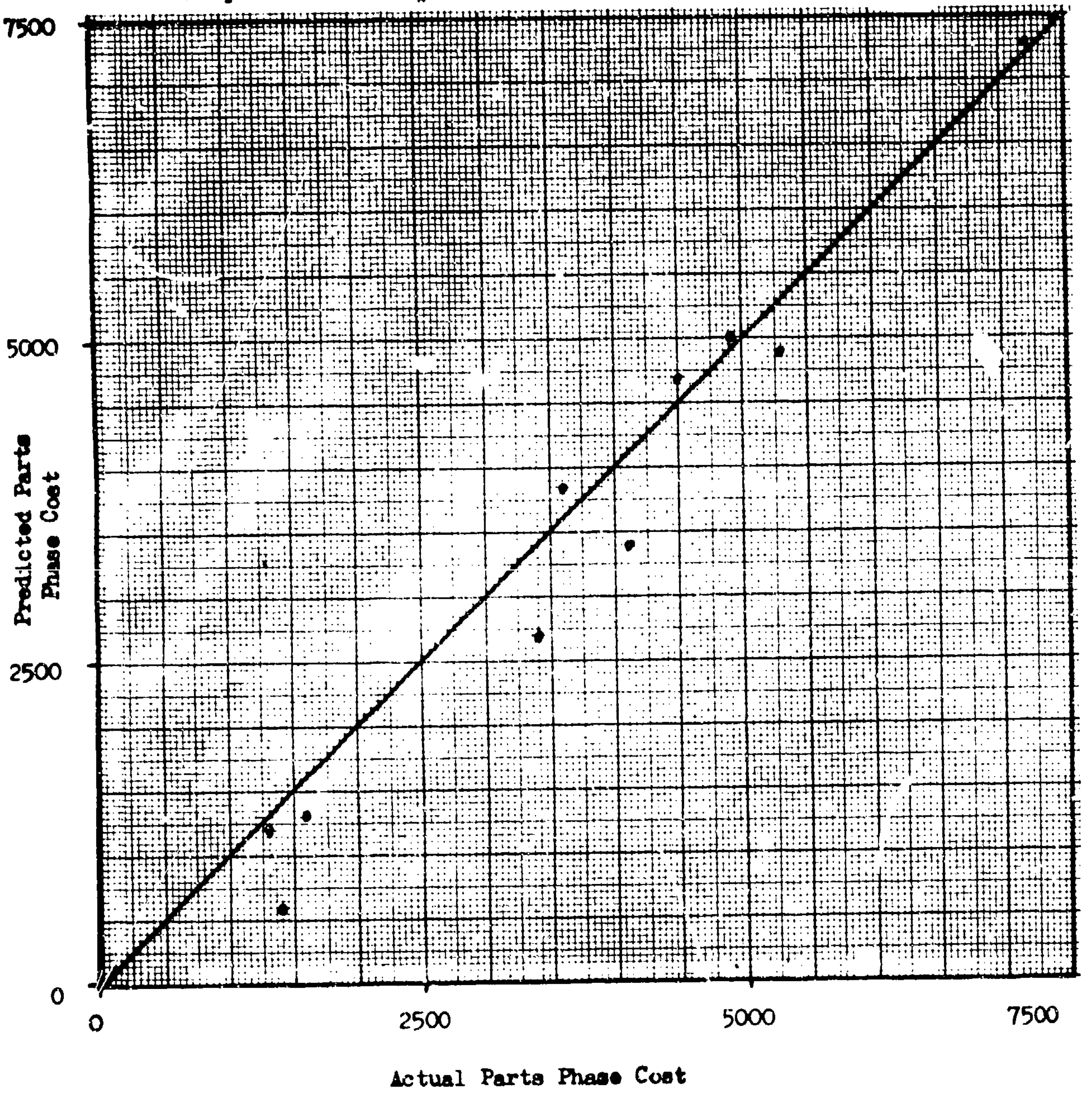


Figure 6.4 Predicted Parts Phase Cost vs Actual Parts Phase Cost

$$C_P = -11{,}603 + 10.08\theta_{pred} - 0.0013\theta^2_{pred} + 0.92N_A - 0.000011N^2_A + 0.52N_D - 0.000021N^2_D$$
$$-0.00031N_A\theta_{pred} - 0.00061N_D\theta_{pred}$$

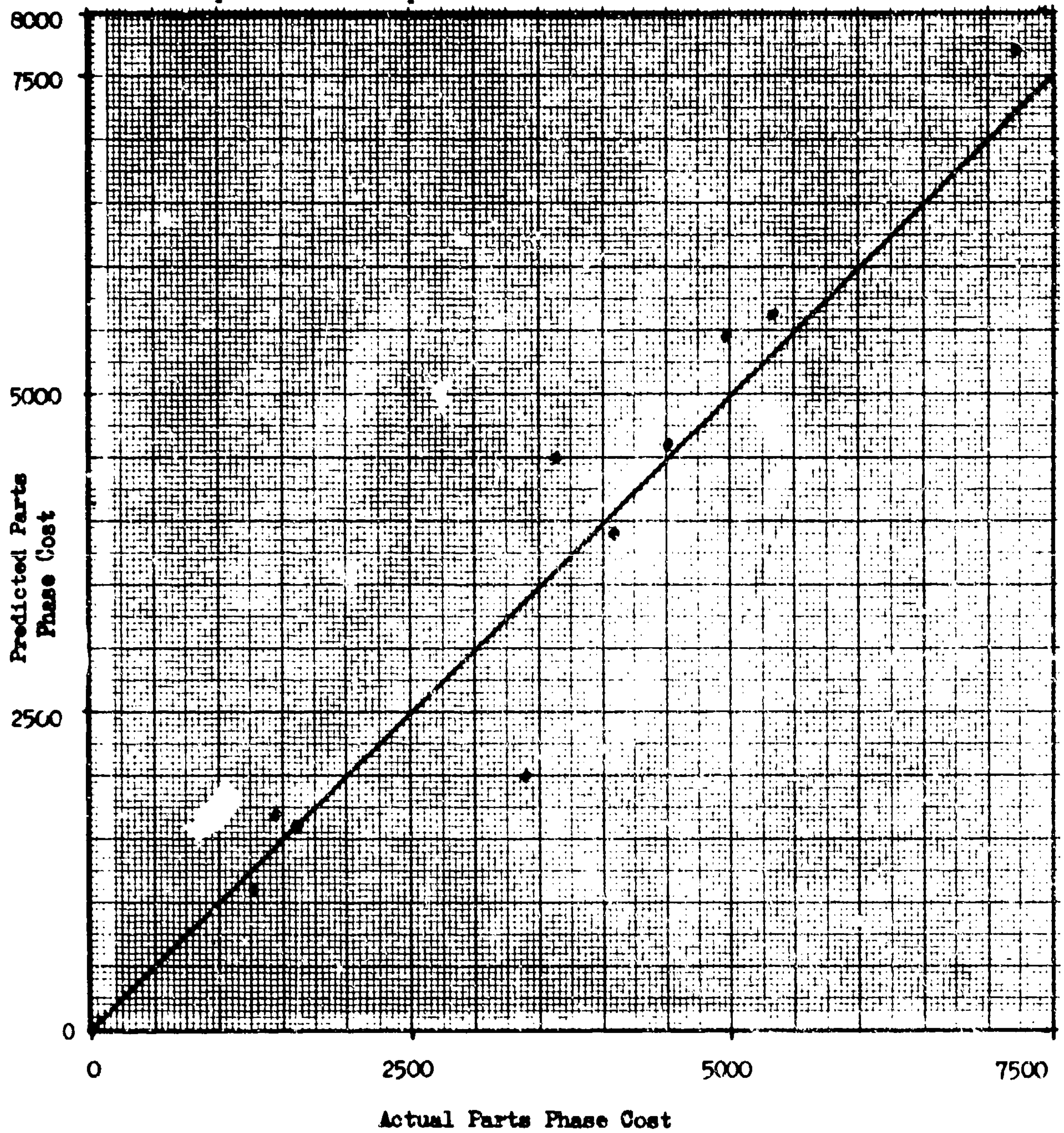


Figure 6.5 Predicted Evaluation Phase Cost vs Actual Evaluation Phase Cost

$$C_E = -17{,}137 + 28.17\Theta_{spec} - 0.0056\Theta^2_{spec} + 0.79N_A - 0.0000048N_A^2 + 1.78N_D - 0.000051N_D^2$$
$$-0.00067N_A\Theta_{spec} - 0.0031N_D\Theta_{spec}$$

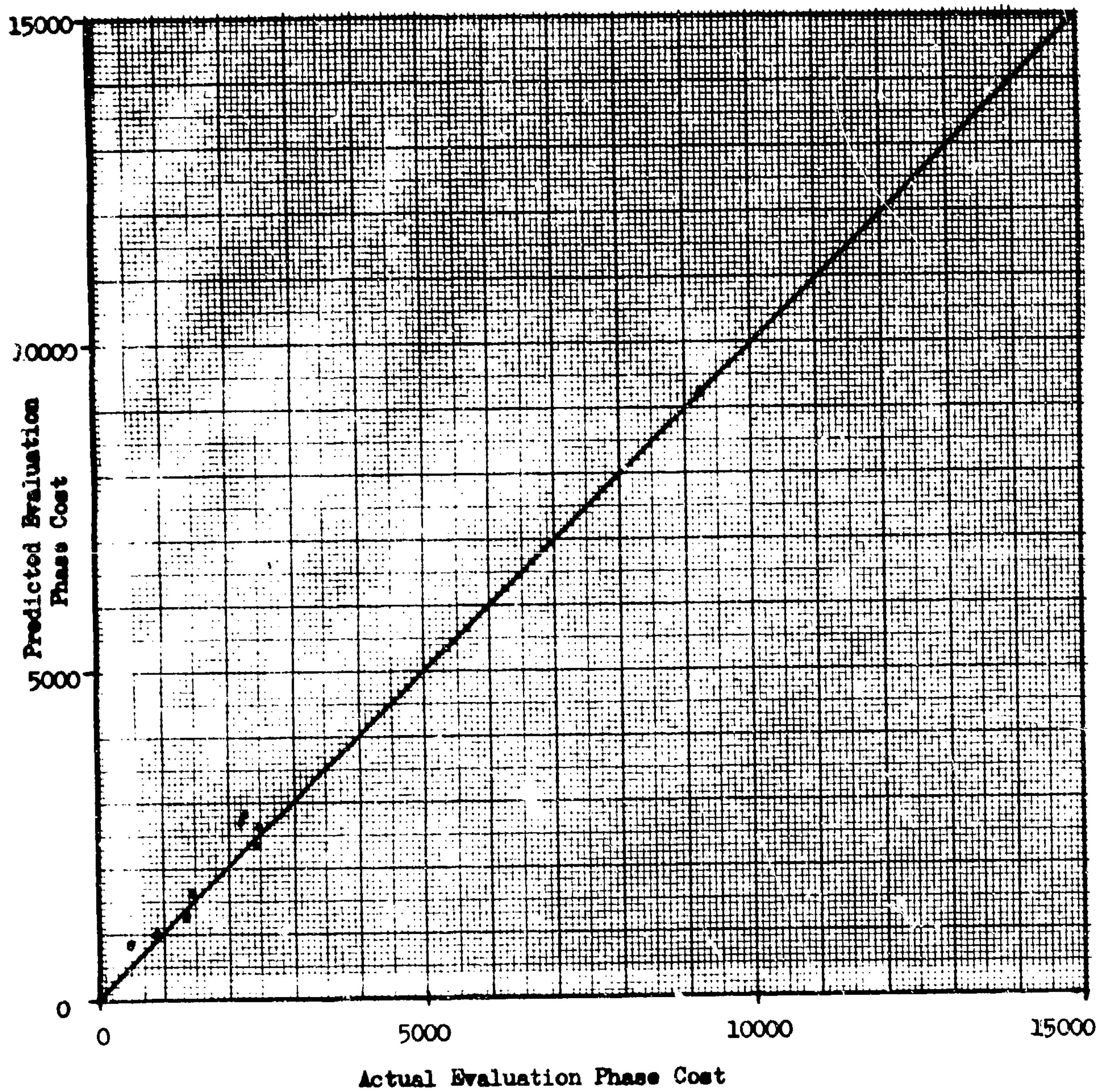


Figure 6.6 Predicted Evaluation Phase Cost vs Actual Evaluation Phase Cost

$$C_E = -18{,}027 + 15.77\Theta_{pred} - 0.0021\Theta^2_{pred} + 1.49N_A - 0.000017N_A^2 + 0.41N_D - 0.000048N_D^2$$
$$-0.00080N_A\Theta_{pred} - 0.00014N_D\Theta_{pred}$$

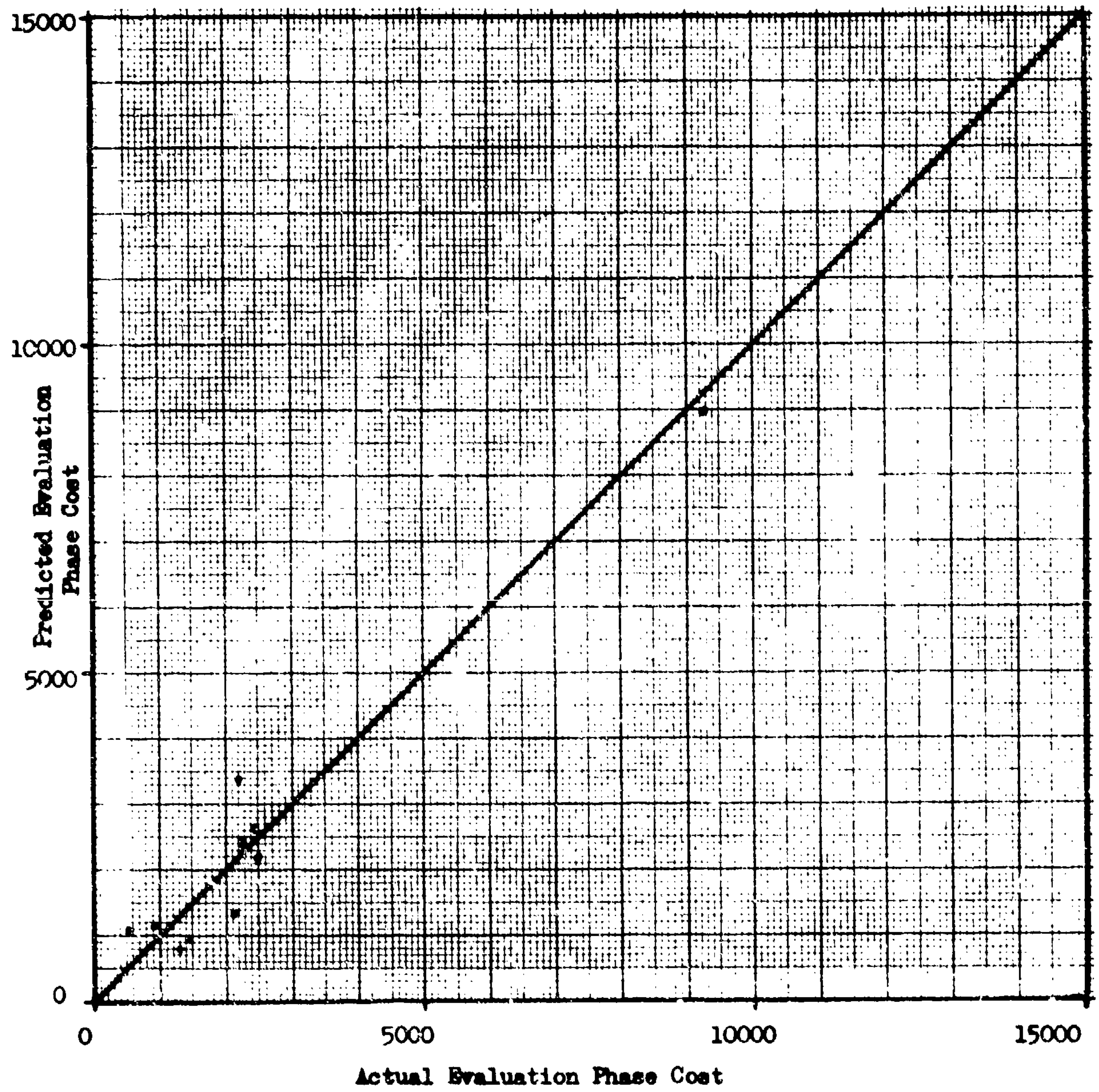


Figure 6.7 Predicted Post Evaluation MTBF vs Actual Post Evaluation MTBF

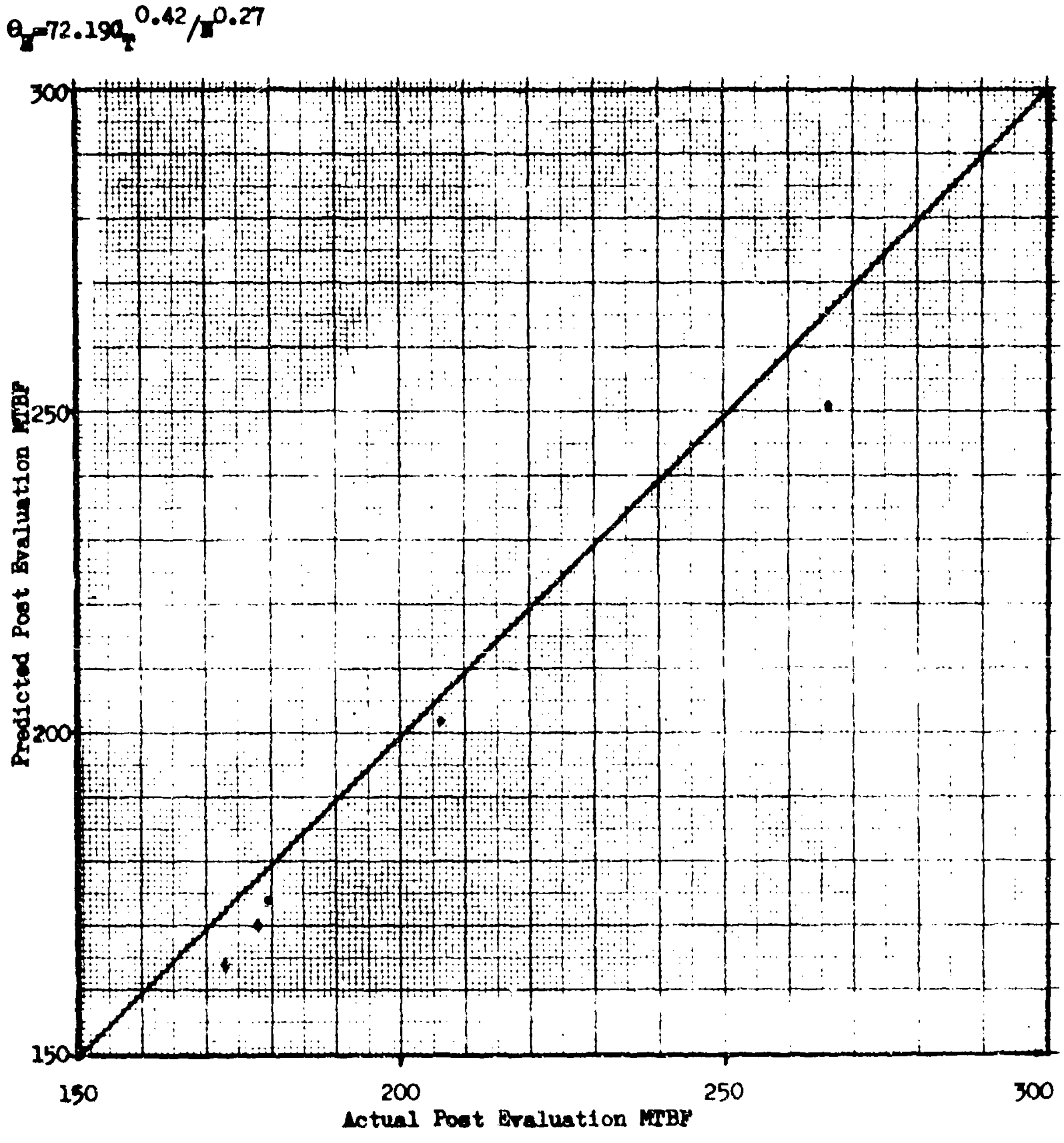


Figure 6.8 Predicted Post Evaluation MTBF vs Actual Post Evaluation MTBF

$\theta_E = 62.95 C_T^{0.44}/(N_A^{0.25} N_D^{0.034})$

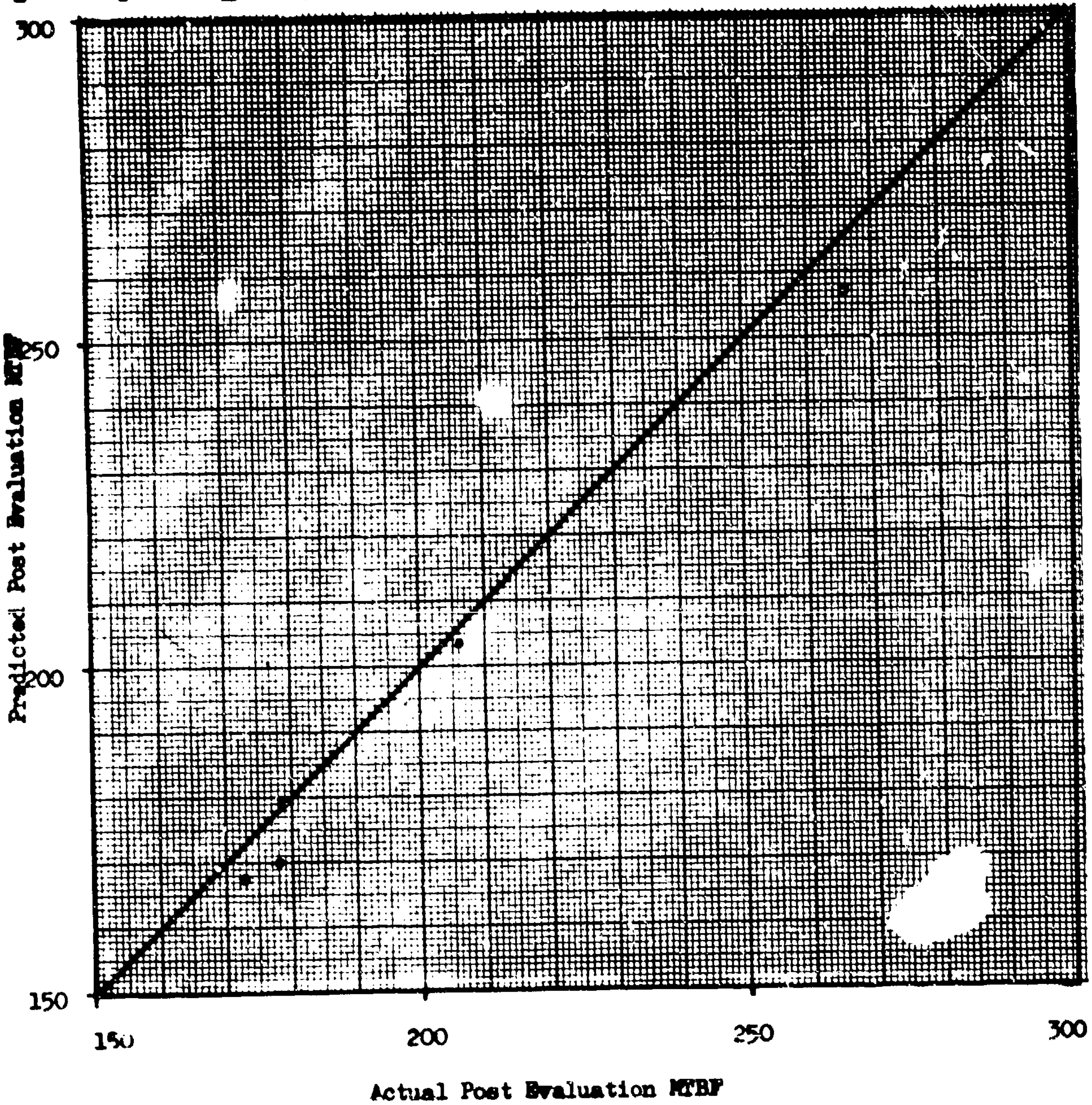


Figure 6.9 Predicted Post Evaluation MTBF vs Actual Post Evaluation MTBF

$\theta_N = 34.91 C_N^{0.28} / N^{0.049}$

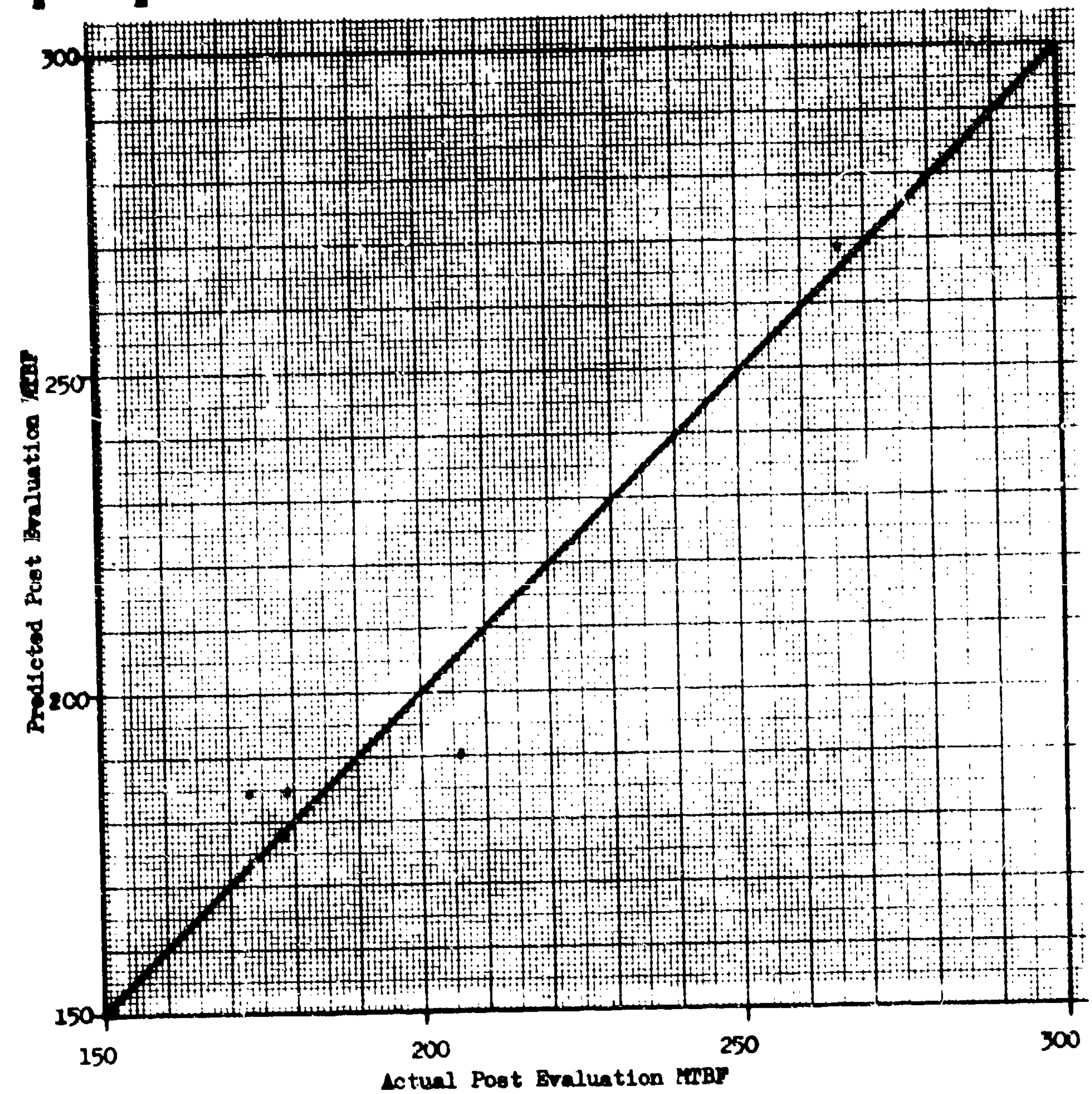


Figure 6.10 Predicted Post Evaluation MTBF vs Actual Post Evaluation MTBF

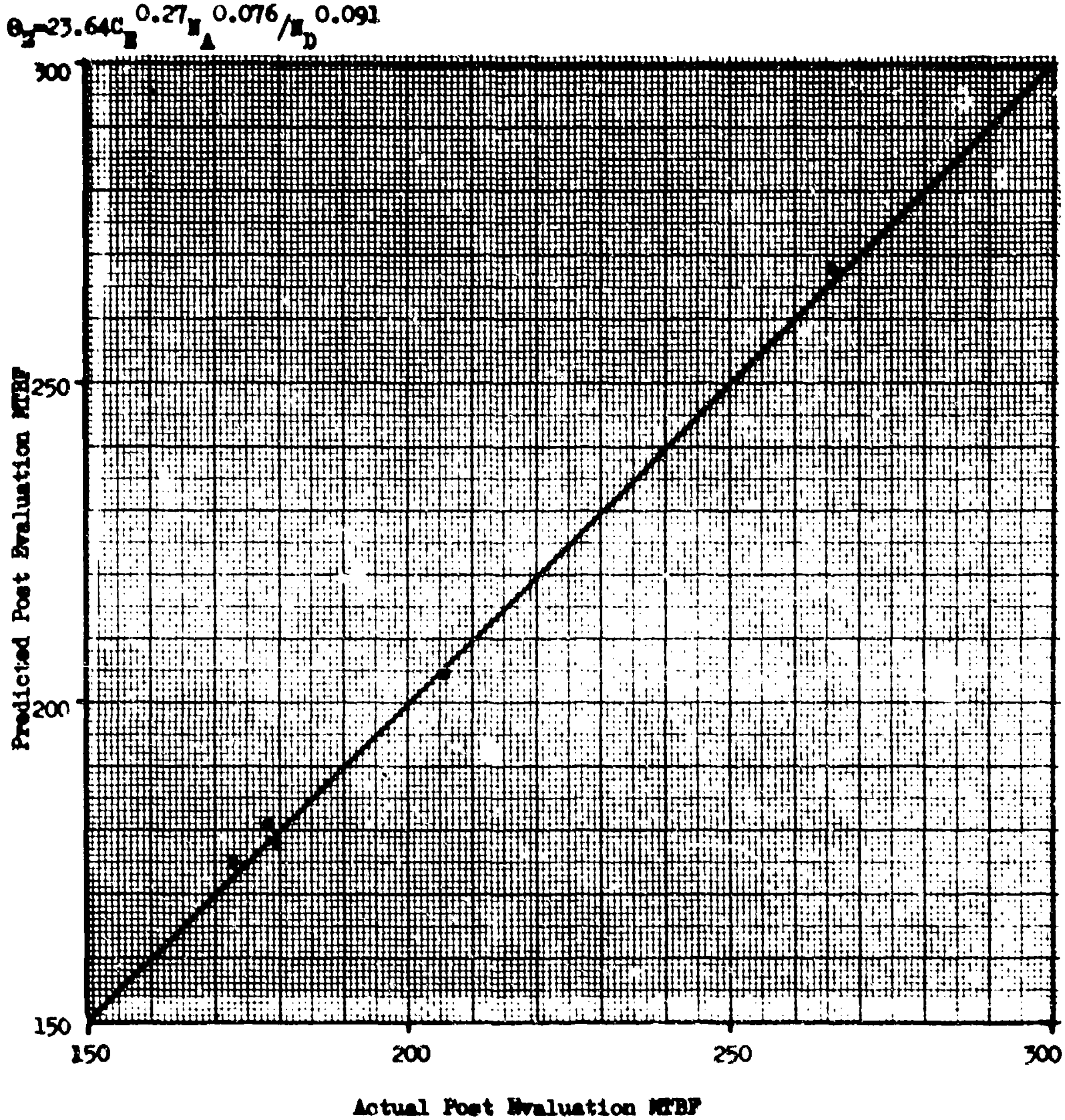


Figure 6.11 Predicted Total Gain vs Actual Total Gain

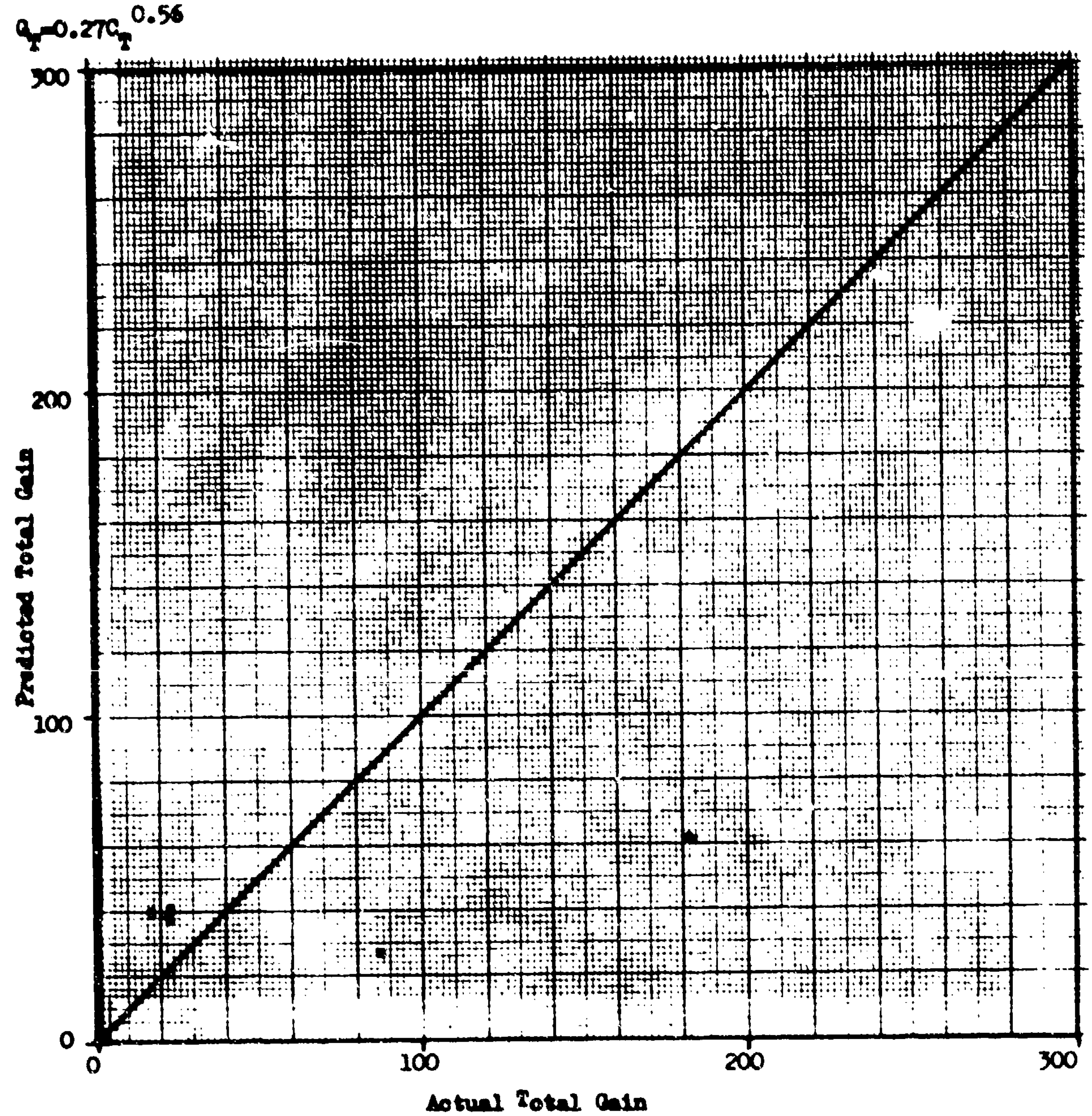


Figure 6.12 Predicted Total Gain vs Actual Total Gain

$G_T = (2.31\times10^{-11})C_D^{6.44}C_P^{-3.61}C_E^{3.01}$

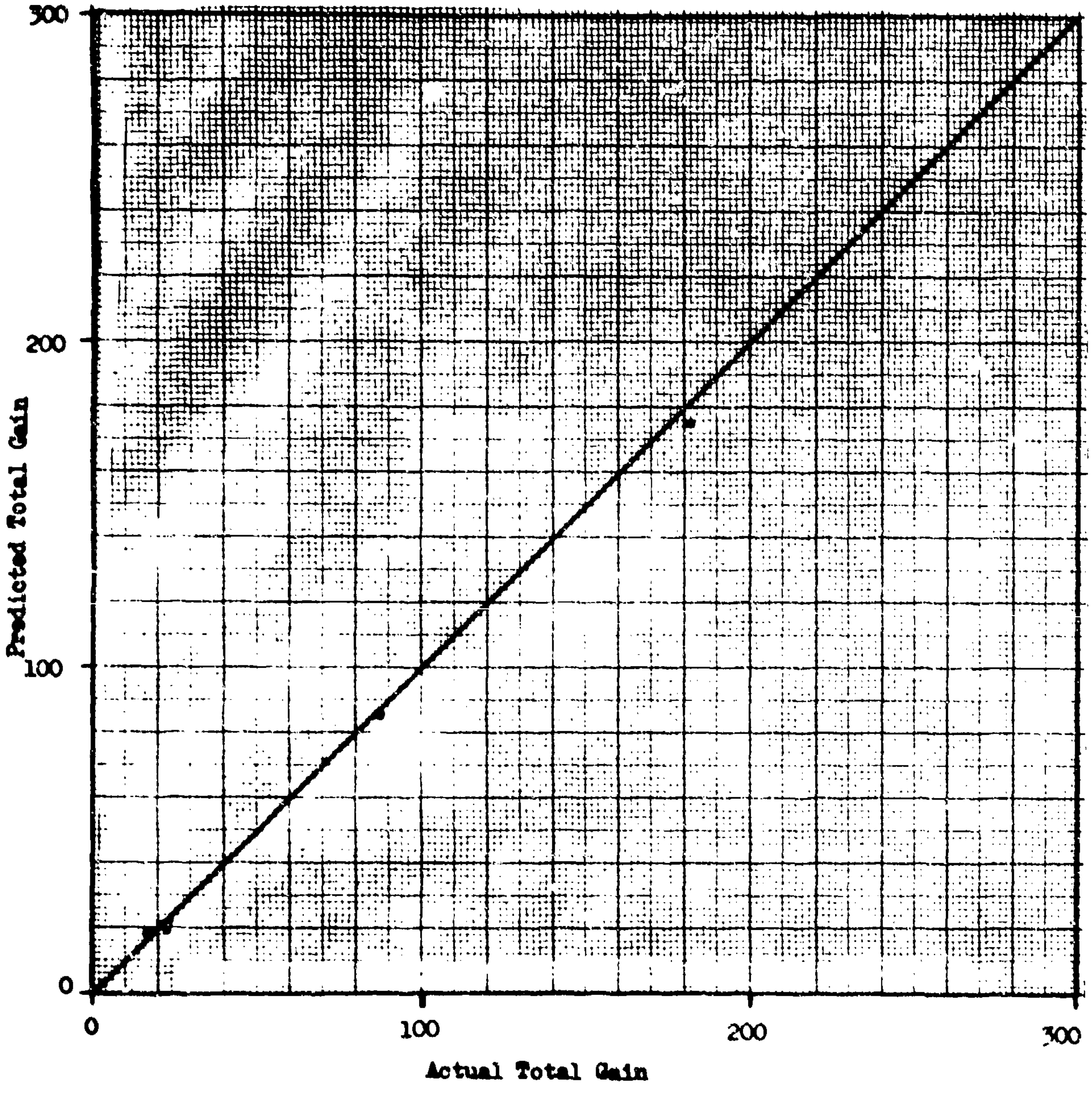


Figure 6.13 Predicted Total Gain vs Actual Total Gain

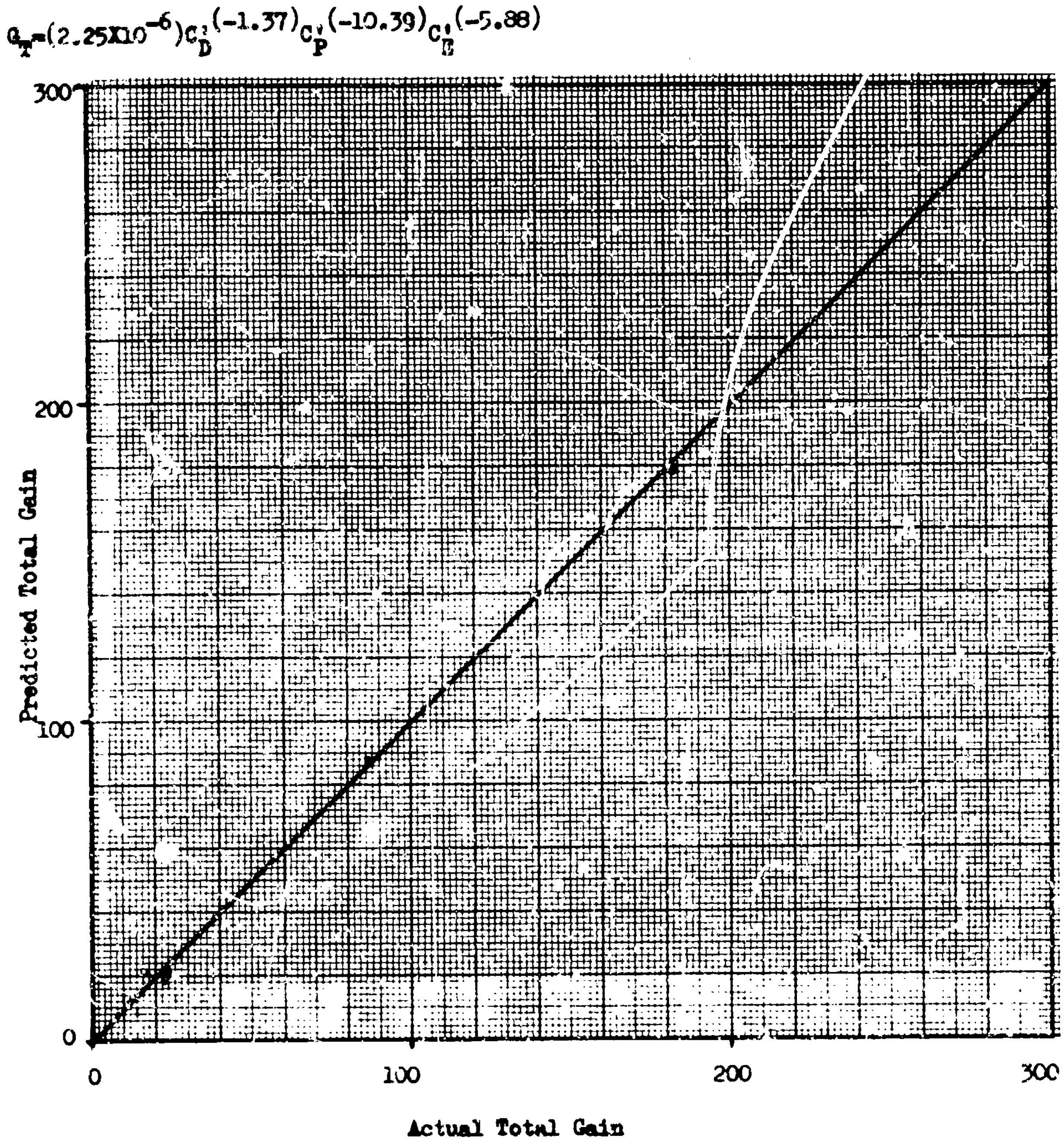


Figure 6.14 Predicted Design Gain vs Actual Design Gain

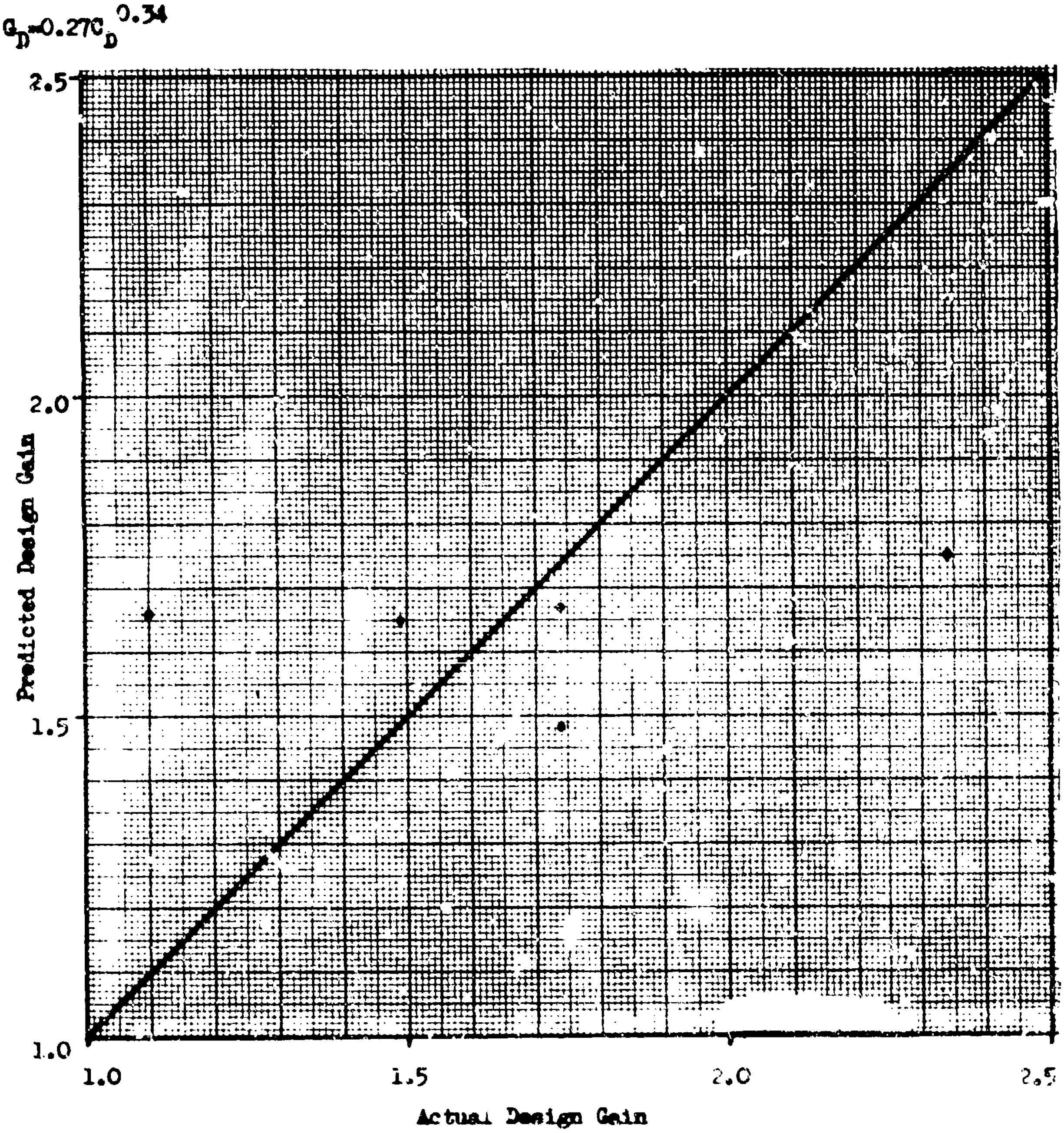


Figure 6.15 Predicted Design Gain vs Actual Design Gain

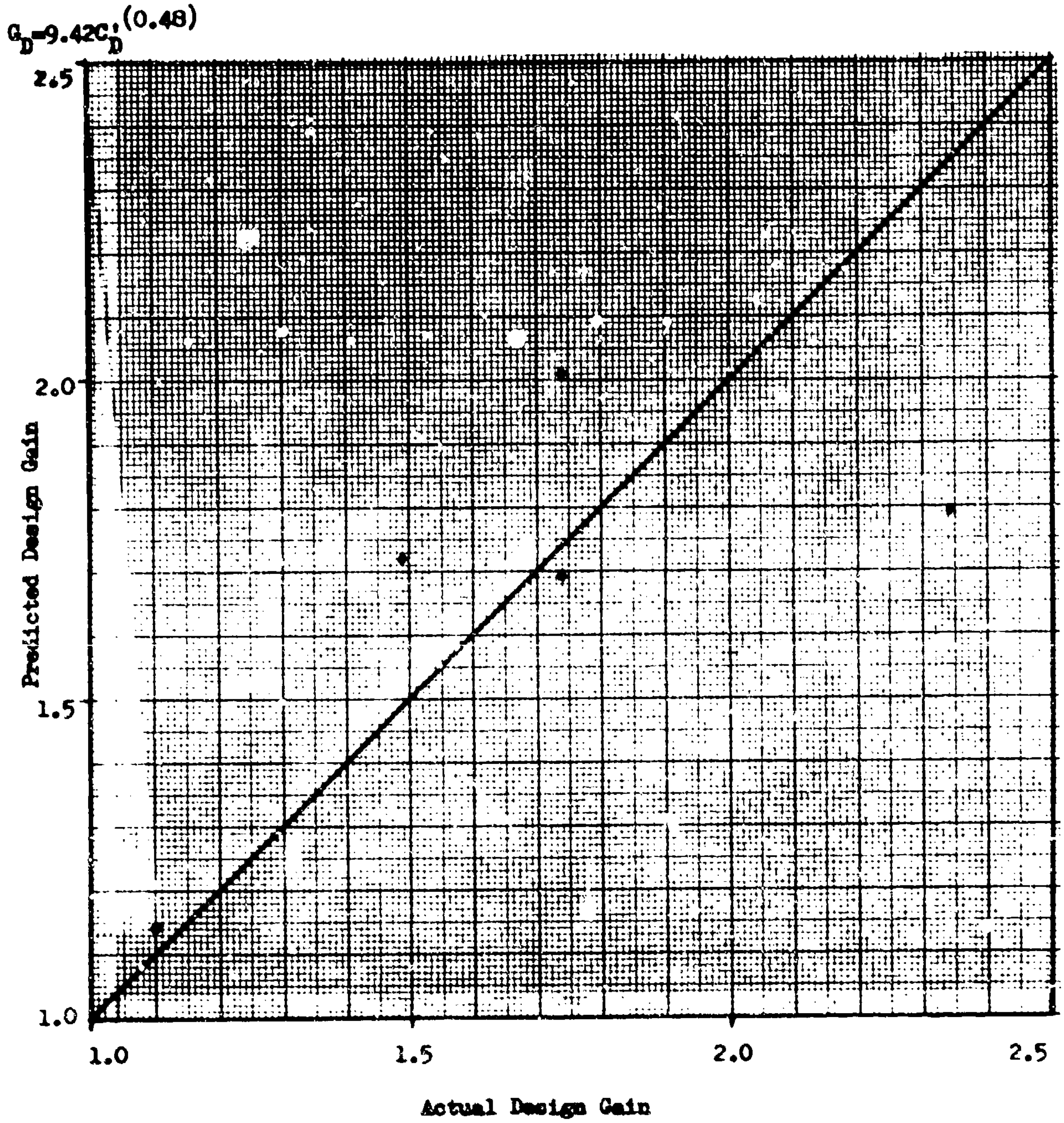


Figure 6.16 Predicted Parts Gain vs Actual Parts Gain

$G_p = 0.19C_p^{0.29}$

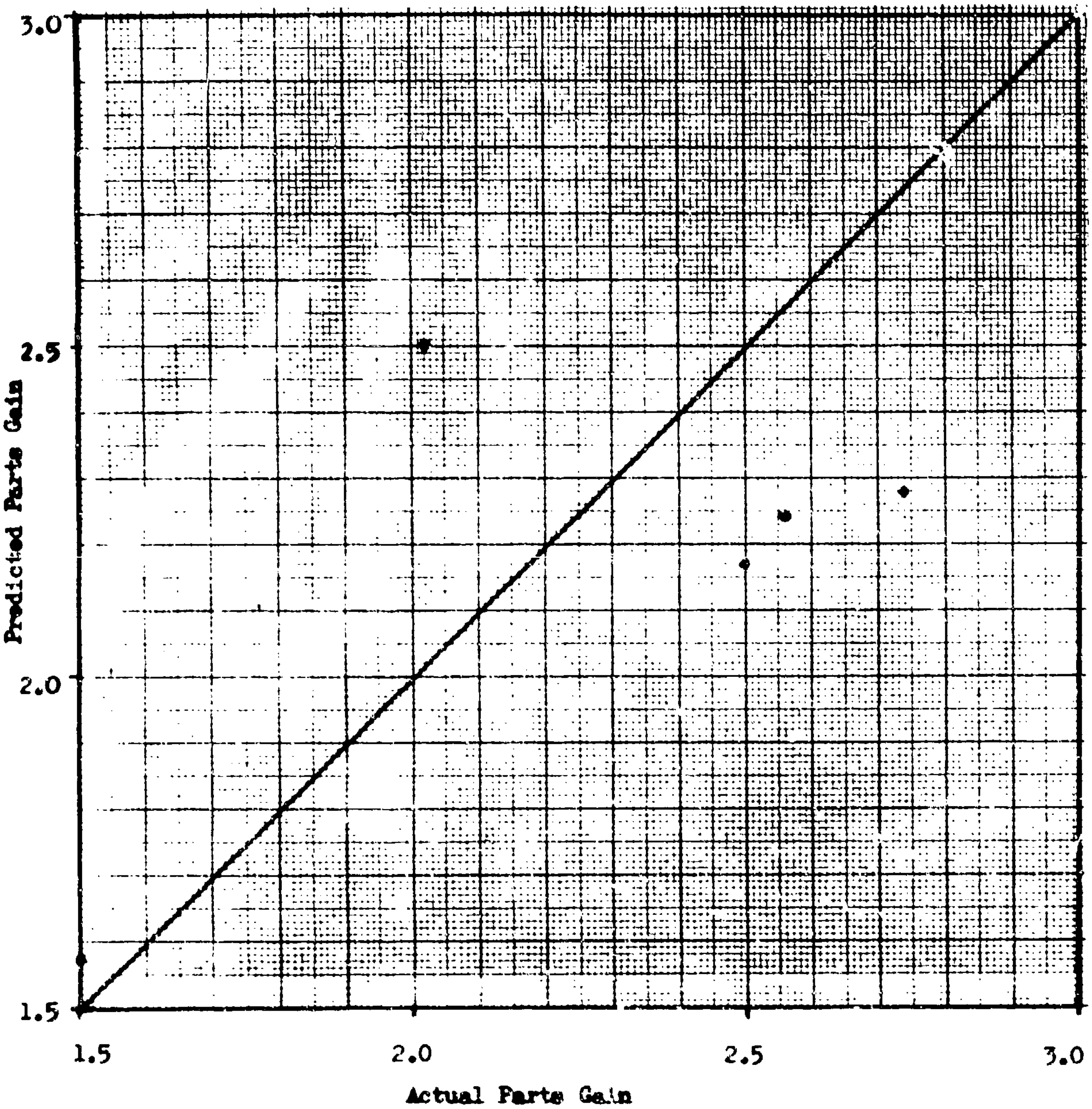


Figure 6.17 Predicted Parts Gain vs Actual Parts Gain

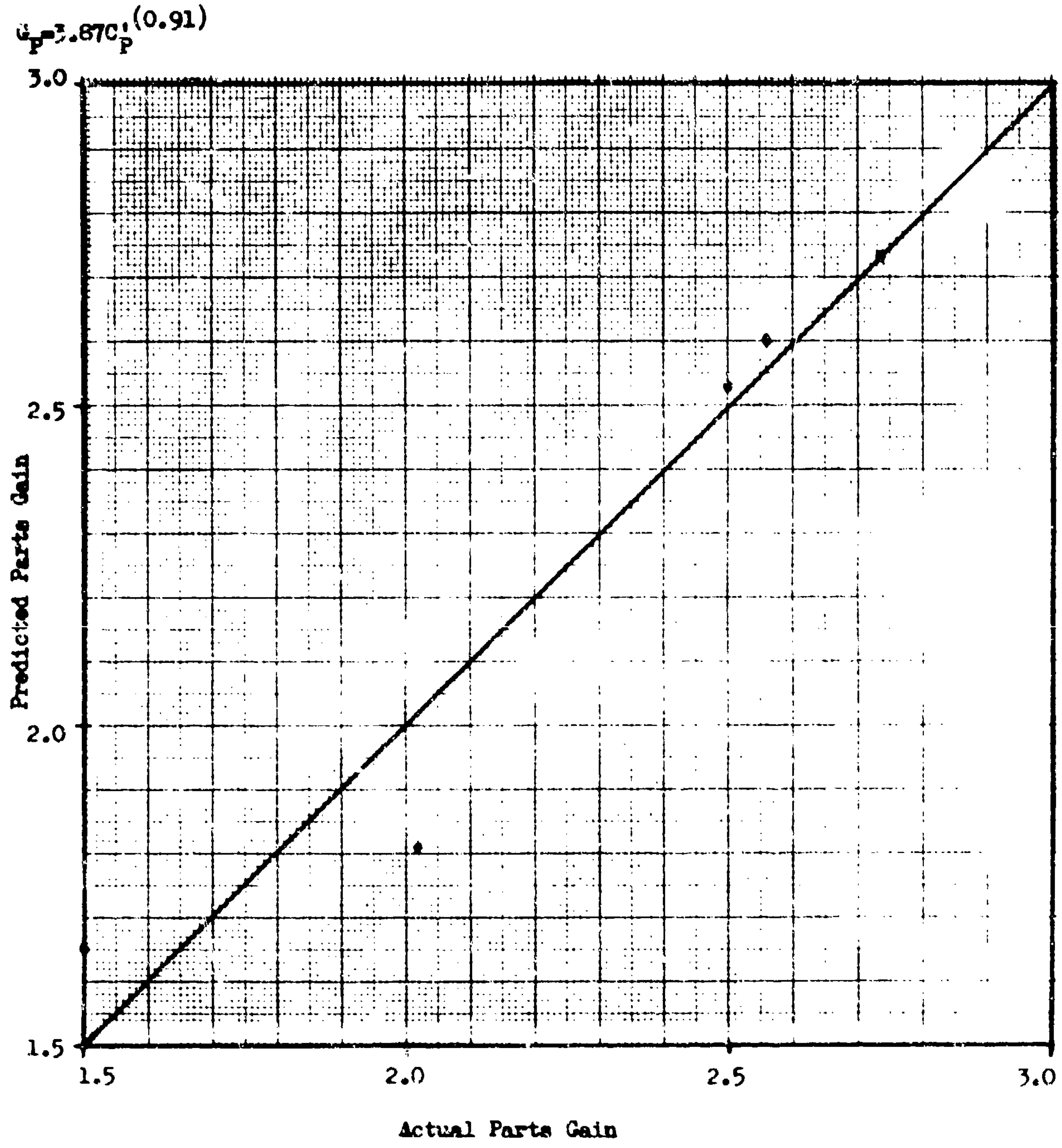


Figure 6.18 Predicted Evaluation Gain vs Actual Evaluation Gain

$G_E = 0.000029 C_E^{1.61}$

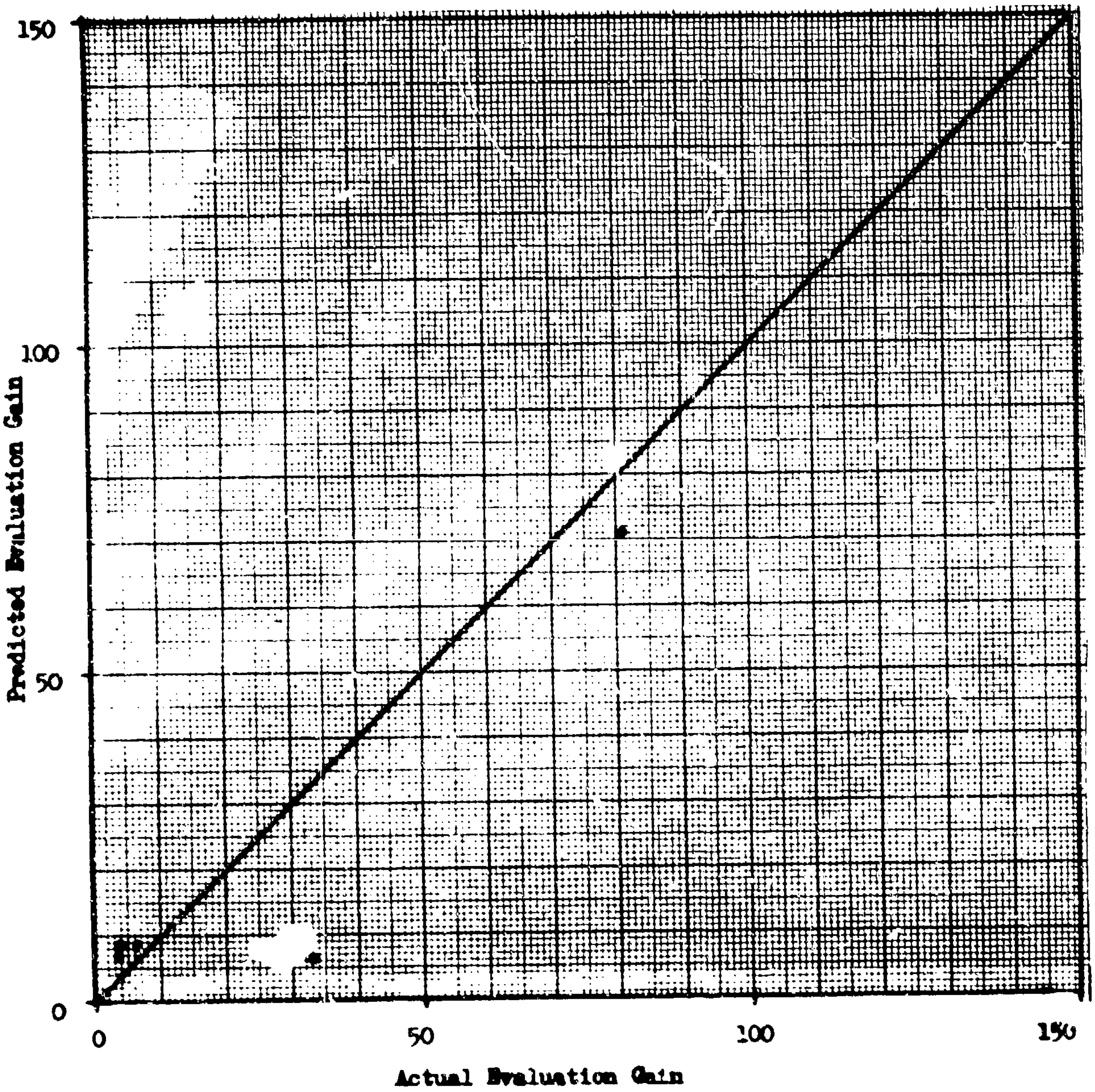
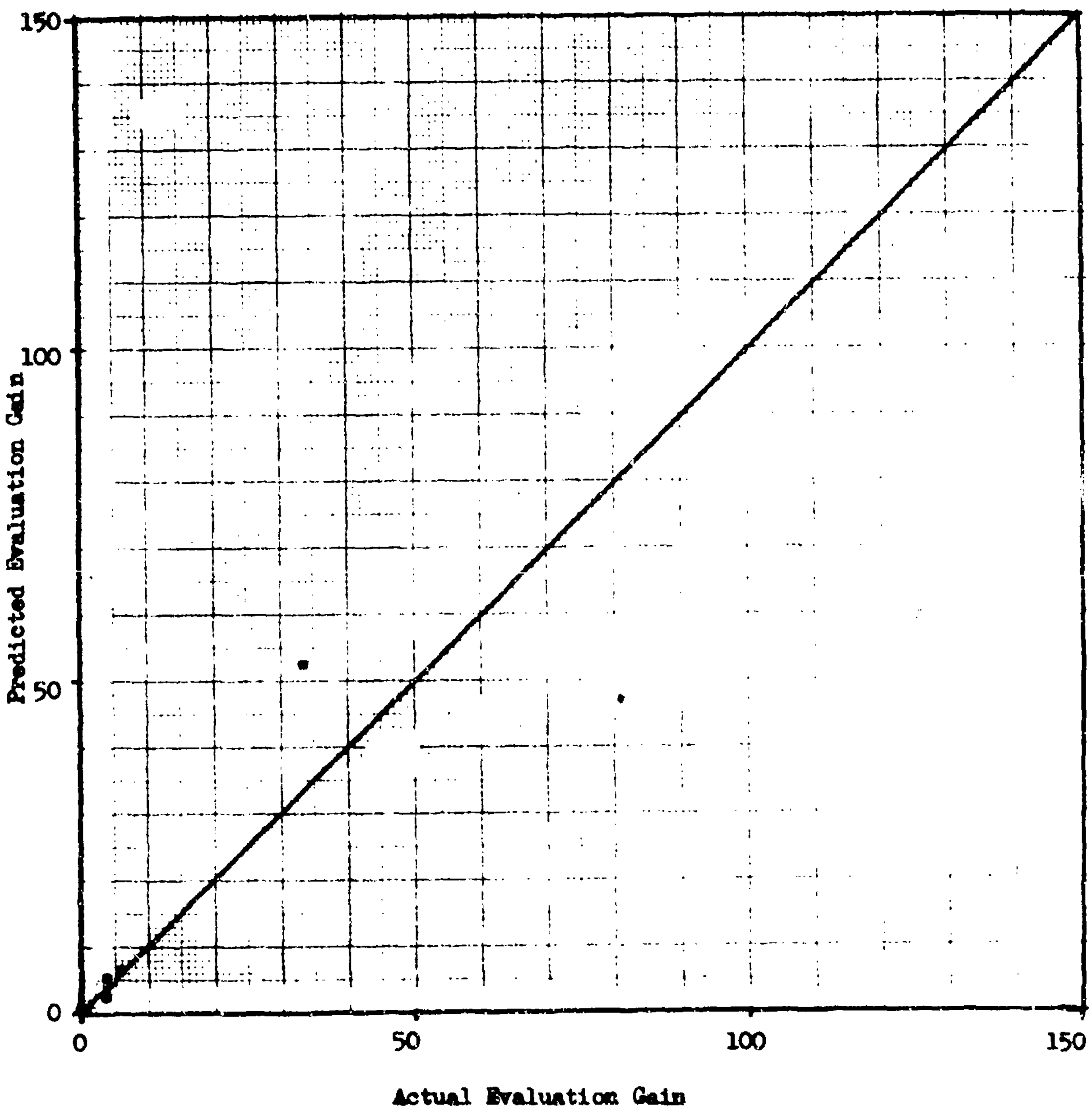


Figure 6.19 Predicted Evaluation Gain vs Actual Evaluation Gain

$G_E = 561.94 C_E'^{(4.20)}$



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